Some theorems about hermitian operators

* Additional comment (I will put this slide and the next two up on the course web page).
Recall we noted above:

Note that the eigenfunctions of hermitian operators corresponding to physically interesting observables are *not* always normalisable. For example:

- The momentum operator $\hat{p} = -i\hbar\frac{\partial}{\partial x}$ has eigenfunctions $\exp(\frac{i}{\hbar}px)$ with eigenvalue p.
- The position operator $\hat{x}$ has eigenfunctions $\delta(x-q)$ with eigenvalue q: we have $\hat{x}\delta(x-q) = x\delta(x-q) = q\delta(x-q)$.

Although we won't worry too much about it in this course, this tells us that we can never measure position or momentum of a quantum particle with perfect precision, even in principle -- if we could then the projection postulate would tell us we would create an unphysical (because unnormalisable) wavefunction.

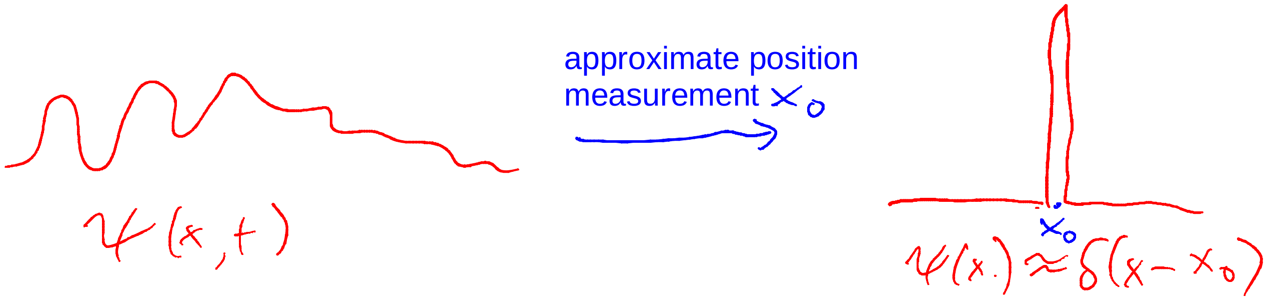
In practice, no real world position measurement has infinite precision:
The Born rule for position measurements is an idealisation.

(Even the finite interval version of the Born rule is an idealisation, though a very useful and good approximation to the actual effect of real world position measurements.)

Quantum measurement postulates

Postulate 4 (the *projection postulate*) If a measurement of the observable O is carried out on a particle with normalised wavefunction $\psi(x, t)$ at time t and the outcome λ_i is obtained, the wavefunction instantaneously after the measurement becomes $\psi_i(x)$.

(This is sometimes referred to as the "collapse of the wavefunction".)



What is really going on in the double slit experiment?

One can try to think of simple models: for example Schrödinger's “charge cloud” picture of the quantum wave function depicted here.

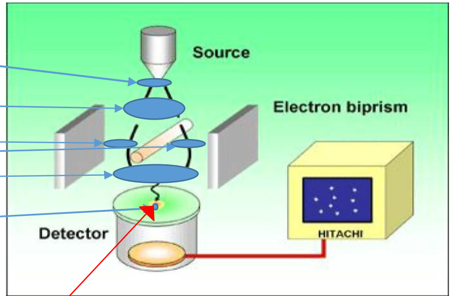
Small dense charge cloud

Bigger less dense charge cloud

Charge cloud separates into two

Two clouds recombine into one

Cloud pulls itself together to become a particle at the detector screen



But these simple models all have flaws. For example, the charge cloud would have to pull itself together with arbitrary large speed, which contradicts Einstein’s relativity theory.

"Collapse" of wave function to (approximate) delta function caused by (approximate) position measurement of detector.

*End of non-examinable comment