Quantum Information and Representation Theory

dedicated to the memory of Graeme Mitchison

Aram Harrow, 13 April 2019

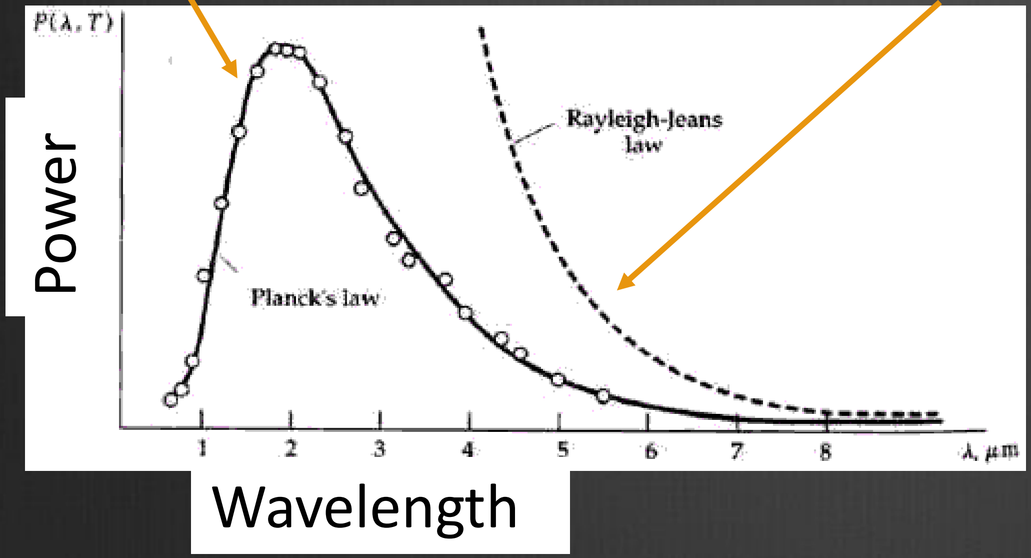
Quantum mechanics

Blackbody radiation paradox:

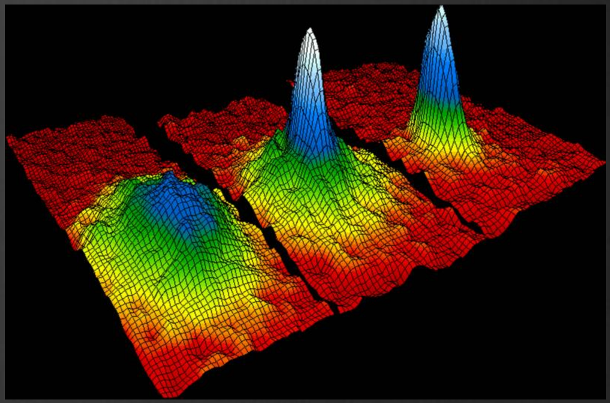
How much power does a hot object emit at a given wavelength ?

Quantum theory
(1900 – 1924)

Classical theory (1900):



Bose-Einstein condensate (1995)

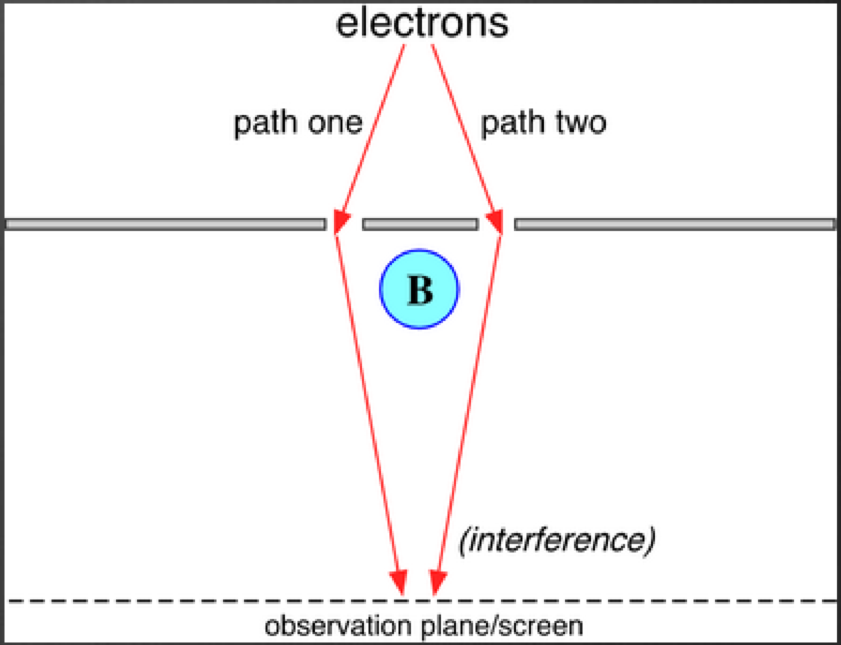


QM has also explained:

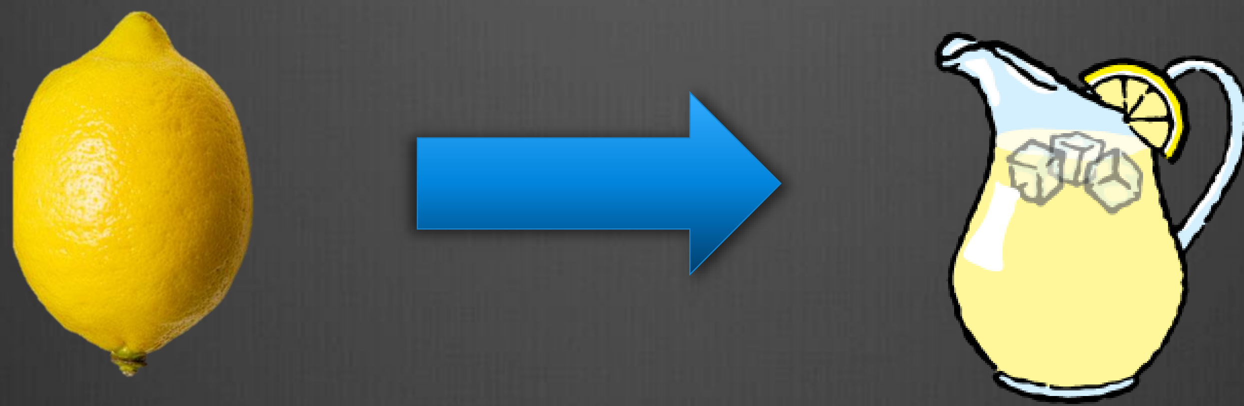
- the stability of atoms
- the photoelectric effect
- everything else we've looked at

Difficulties of quantum mechanics

- Heisenberg's uncertainty principle
- Topological effects
- Entanglement
- Exponential complexity:
  Simulating N objects requires effort »2^N



The doctrine of quantum information



- Abstract away physics to device-independent fundamentals: “qubits”
- Understand foundations through operational statements. By asking “what can we do with quantum information” we learn “what is quantum information.”

Classical information theory

Understand statistics of strings from a small alphabet.

- 11100000110100111100111101100110010111111010110101
- TCCGGAAGTCACAGTTTCAATCCCACTGATCGATGCT
- CLAUDE_SHANNON_A_MATHEMATICAL_THEORY_OF_COMM...

Questions include

- How much information is carried per symbol?
- How to quantify correlations?
- When are errors correctable?

Applications not only to information technology but also to understanding language, genomics, statistics, thermodynamics,...

Quantum information

bit
0 or 1

probability
distribution $\begin{pmatrix} p_0 \\ p_1 \end{pmatrix}$

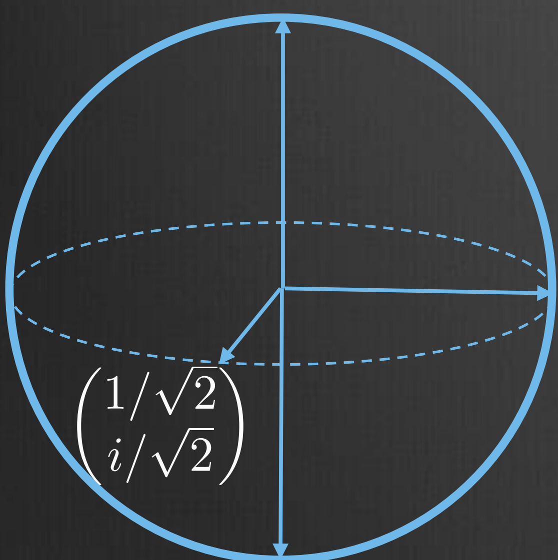
e.g. $\begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix}$

quantum
superposition $\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$

complex numbers
with $|\alpha_0|^2 + |\alpha_1|^2 = 1$

e.g. $\begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \end{pmatrix}$

electron spin



applications include

- secret communication
- distributed quantum computing
- clock synchronization

representation theory

study of symmetry and transformations

example: rotations by 0, 90, 180 or 270 degrees

1	1
1	1

rotations do nothing

1	-1
-1	1

rotations by 0 or 180 do nothing
rotations by 90 or 270 multiply by -1

0	1
1	0

rotations do not just act by multiplying,
but we can relate to irreps (irreducible representations)

0	1
1	0

= ½

1	1
1	1

– ½

1	-1
-1	1

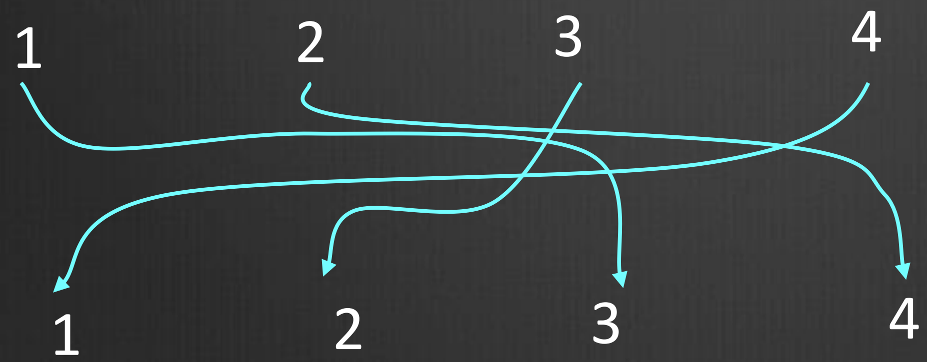
representation theory

example: unit analysis

How do quantities like meters, miles/gallon,
Joules ($kg \cdot m^2 \cdot s^{-2}$), frequency ($s^{-1}$), ...
change when we change our units of space or time?

nonabelian transformations

S_n = permutations of n objects, e.g.



irrep action is more than just multiplying but decomposition still works.

symmetries in information theory

babcbbaacba

- S_n permutes the n=11 symbols
  → type (4,5,2)

- S_d permutes the alphabet of size d=3
  → sorted type (5,4,2)

- Sorted types characterize entropy.
  Not many strings have sorted type (11,0,0).
  Lots of strings have sorted type (4,4,3).

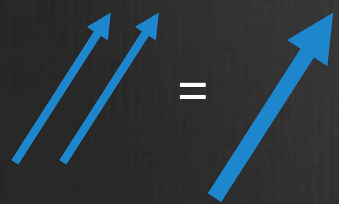
- Quantum states are acted on by S_n and
  U_d = rotations of d-dimensional complex space.
  Schur-Weyl duality relates these actions.

Schur-Weyl duality for two qubits

S_2 = {id, SWAP}
irreps = {symmetric, antisymmetric}

U_2 = rotations of 2-d complex space
≈ rotations of 3-d real space

irreps describe how much state changes under rotation



spins pointing the same way
are sensitive to rotation

spins pointing the opposite way
are insensitive to rotation

This connection is behind ferromagnetism.

A sample of Graeme's work

- Used representation theory to estimate the entropy of quantum states.
- Used Schur-Weyl duality to connect the quantum marginal problem with the Kronecker coefficients from the representation theory of S_n.
- Several creative approaches to de Finetti theorems, relating symmetry to independence.