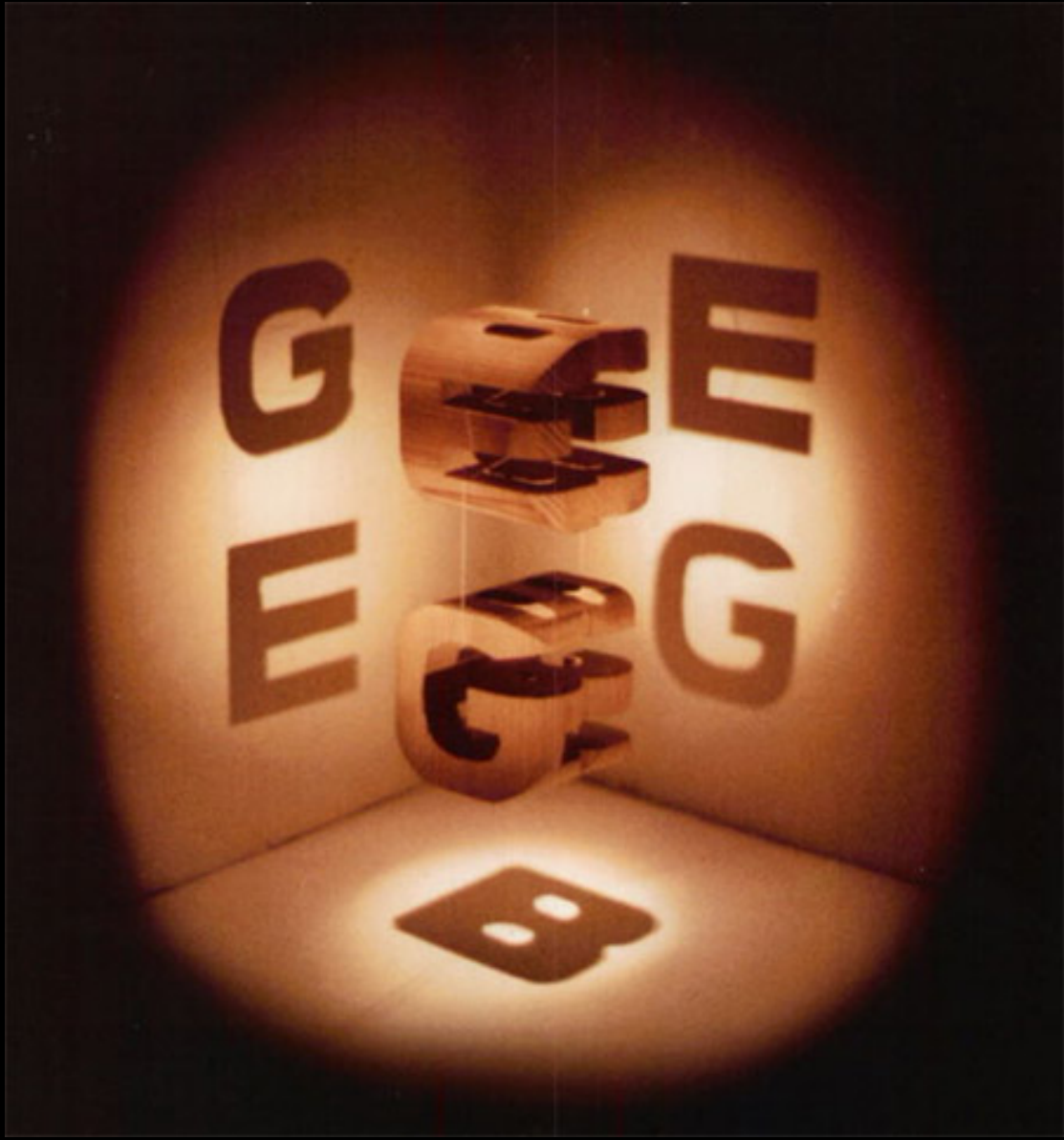


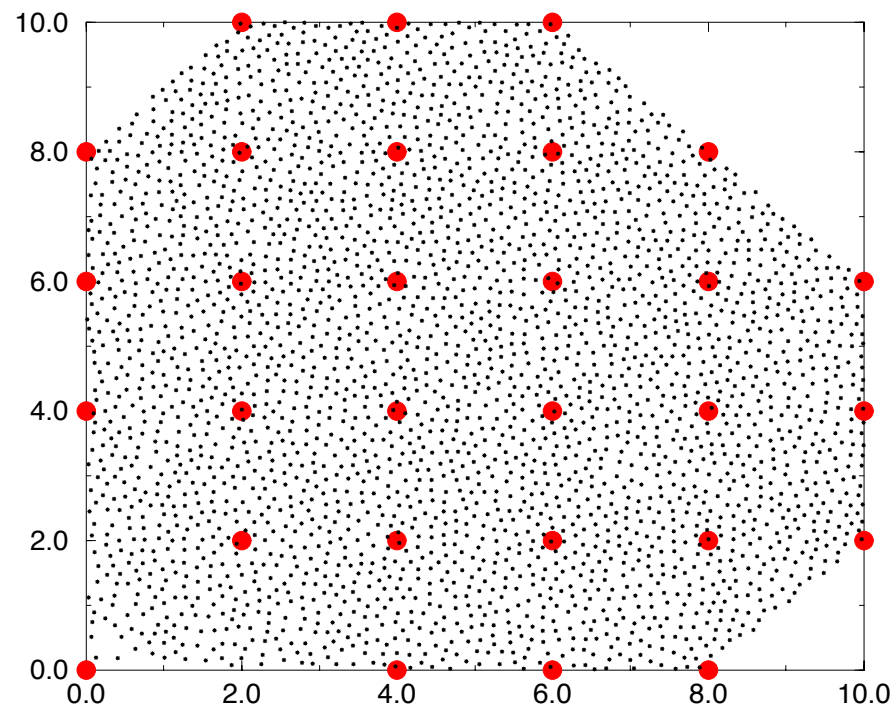
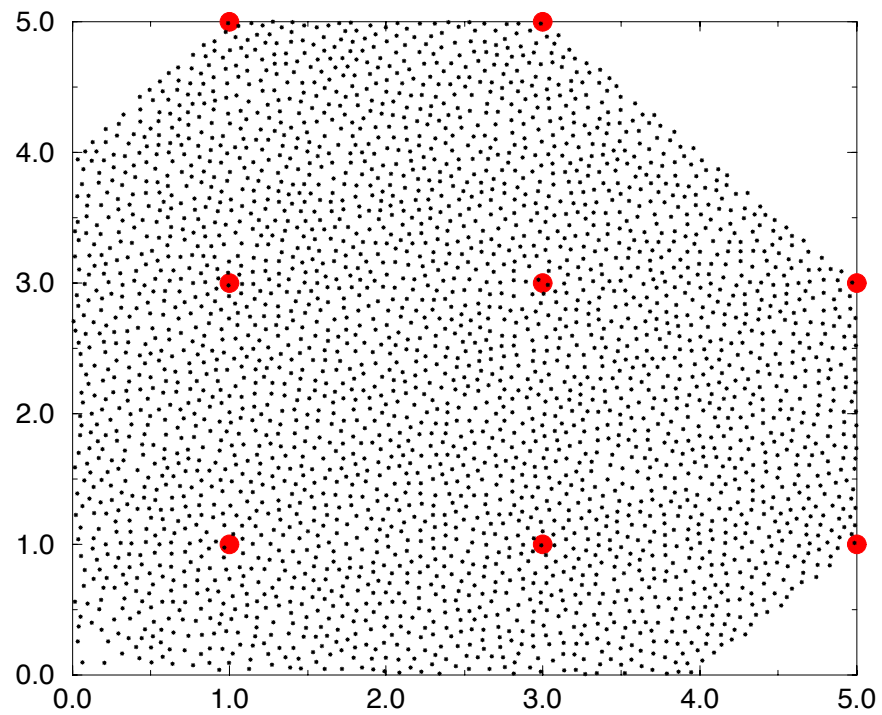
Gödel, Escher, Bach

Matthias Christandl, University of Copenhagen

In memory of Graeme Mitchison









Convexity, Analyticity and Graeme's method

Some preliminaries:

- Permutations will be denoted by π 's so as to reserve g for $SU(d)$ elements
- I will use lowercase p for probabilities, uppercase P for projectors, and the notation $p(\lambda, \mu, \nu, \dots)$ for the joint probability distributions

$$p(\lambda, \mu, \nu, \dots) = \text{Tr}_{A^{\otimes n} B^{\otimes n} C^{\otimes n} \dots} (P_\lambda \otimes P_\mu \otimes P_\nu \dots \rho_{ABC\dots}^{\otimes n}) .$$

- Similarly, the Laplace transform of the p 's will be denoted by $Z(\alpha, \beta, \gamma \dots)$ so that

$$Z(\alpha, \beta, \gamma \dots) = \sum_{\lambda \mu \nu \dots} p(\lambda, \mu, \nu, \dots) e^{-\alpha \cdot \lambda - \beta \cdot \mu - \gamma \cdot \nu \dots} ,$$

where $\alpha \cdot \lambda = \sum_{i=1}^d \alpha_i \lambda_i$.

- For rate functions, I will use ϕ and ζ so that $p \sim e^{-n\phi}$ and $Z \sim e^{-n\zeta}$. More precisely,

$$\phi(\bar{\lambda}, \bar{\mu}, \bar{\nu} \dots) \equiv - \lim_{n \rightarrow \infty} \frac{1}{n} \log p(n\bar{\lambda}, n\bar{\mu}, n\bar{\nu} \dots)$$

$$\zeta(\alpha, \beta, \gamma \dots) \equiv - \lim_{n \rightarrow \infty} \frac{1}{n} \log Z(\alpha, \beta, \gamma \dots) .$$

