de Finetti theorems for symmetric quantum states

Biology, Mathematics and Quantum Information:

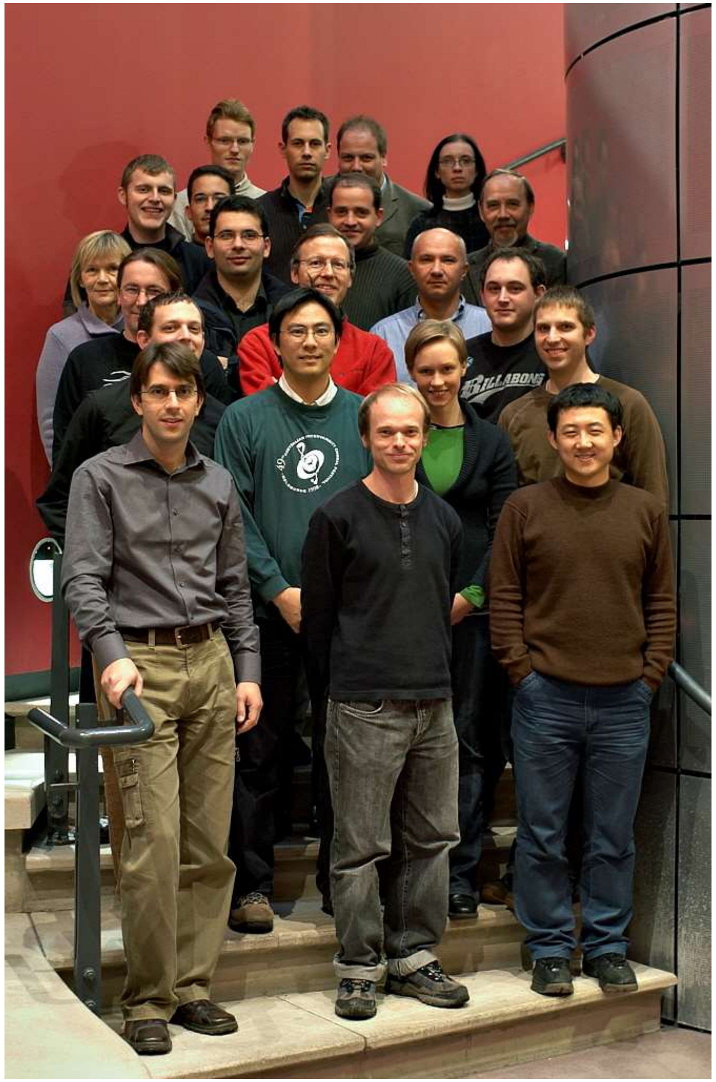
A symposium in memory of Graeme Mitchison (1944 - 2018)

Cambridge, April 13, 2019

Robert Koenig

Centre for Quantum Computation in 2003

Cambridge around 2005



The Cambridge de Finetti'ists

Matthias Christandl

Graeme Mitchison

Renato Renner

Robert König

sketch paper Inbox ×

Graeme J Mitchison <G.J.Mitchison@damtp.cam.ac.uk> Dec 13, 2005, 1:58 PM

to Matthias, Renato, Robert

Dear fellow de Finetti'ists,

Here is a paper I have sketched based on what we have done so far.

•••

Graeme

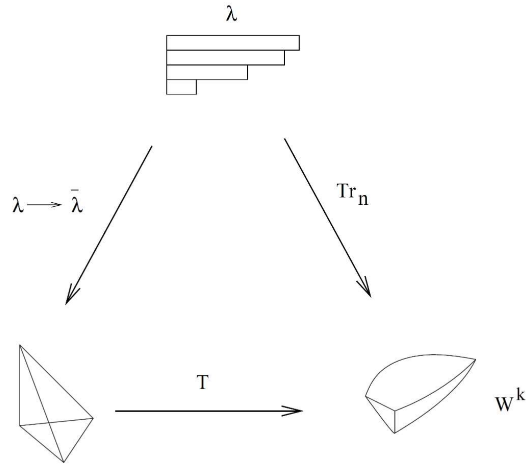


Figure 1: This shows schematically the maps underlying the main theorem. From the Young diagram λ on $n+k$ systems, one can go to W^k by via tracing out n systems, or one can go to $\Delta(d)$ by normalising the row lengths of λ. From the latter space, the map T takes one to W^k, and the two routes end up at the same point, up to an error of order k/n.

From a discussion between Graeme Mitchison and John Conway

Photo by Robert Dryja

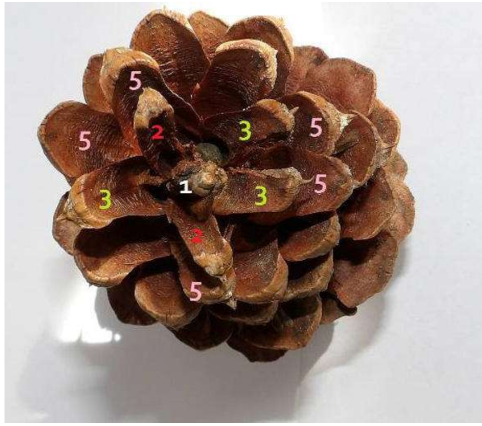


Photo by Robert Dryja

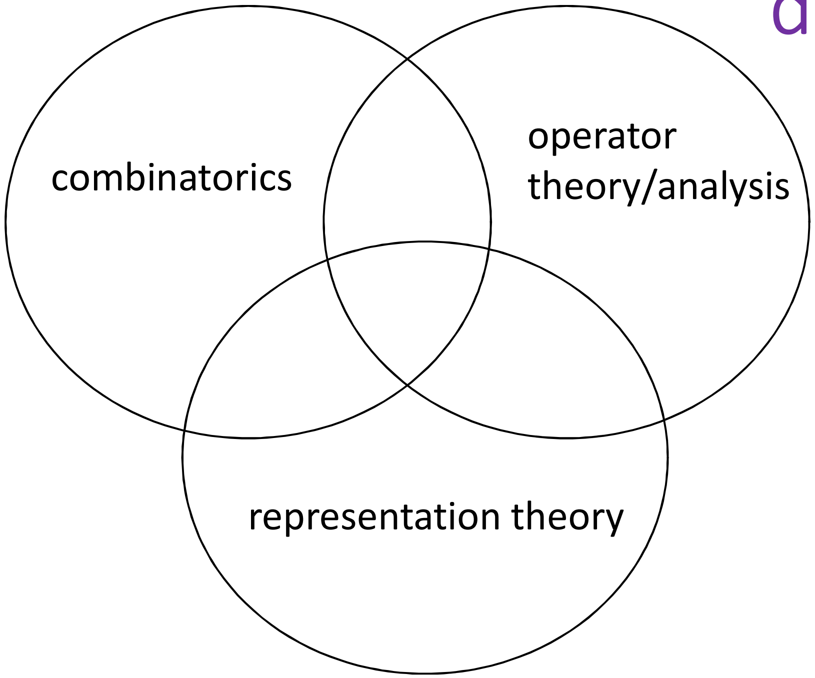
Wythoff array

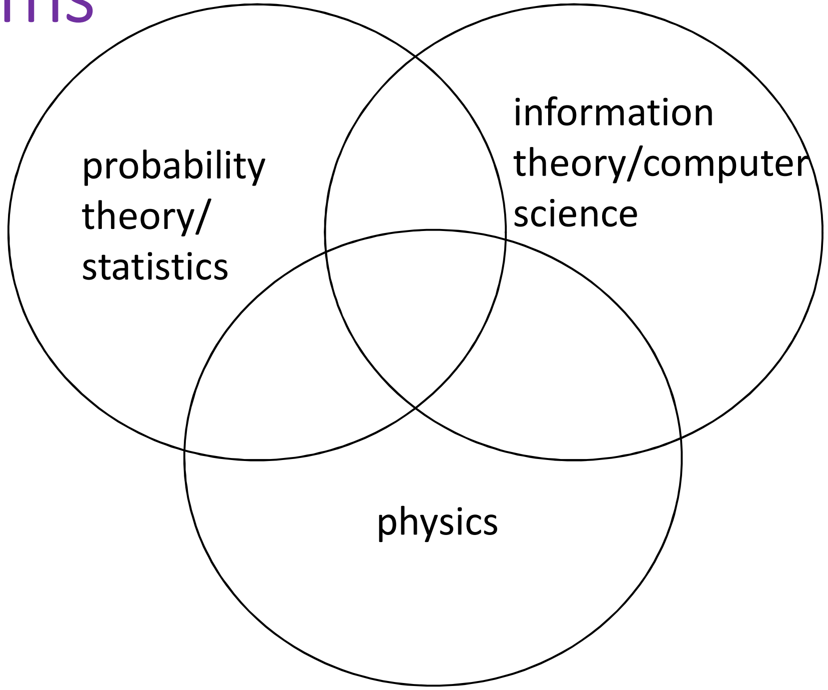
1	2	3	5	8	13	21	⋯
4	7	11	18	29	47	76	⋯
6	10	16	26	42	68	110	⋯
9	15	24	39	63	102	165	⋯
12	20	32	52	84	136	220	⋯
14	23	37	60	97	157	254	⋯
17	28	45	73	118	191	309	⋯
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱

Morrison, D. R. (1980), "A Stolarsky array of Wythoff pairs", A Collection of Manuscripts Related to the Fibonacci Sequence(PDF), Santa Clara, Calif: The Fibonacci Association, pp. 134–136.

https://www.ladailypost.com/content/amateur-naturalist-mathematical-symmetry-pine-cones

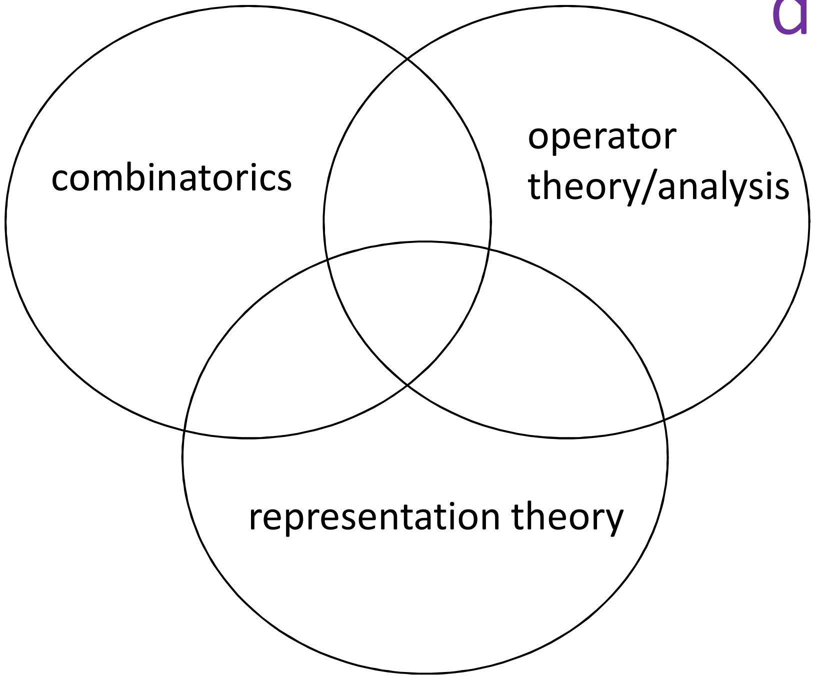
de Finetti theorems

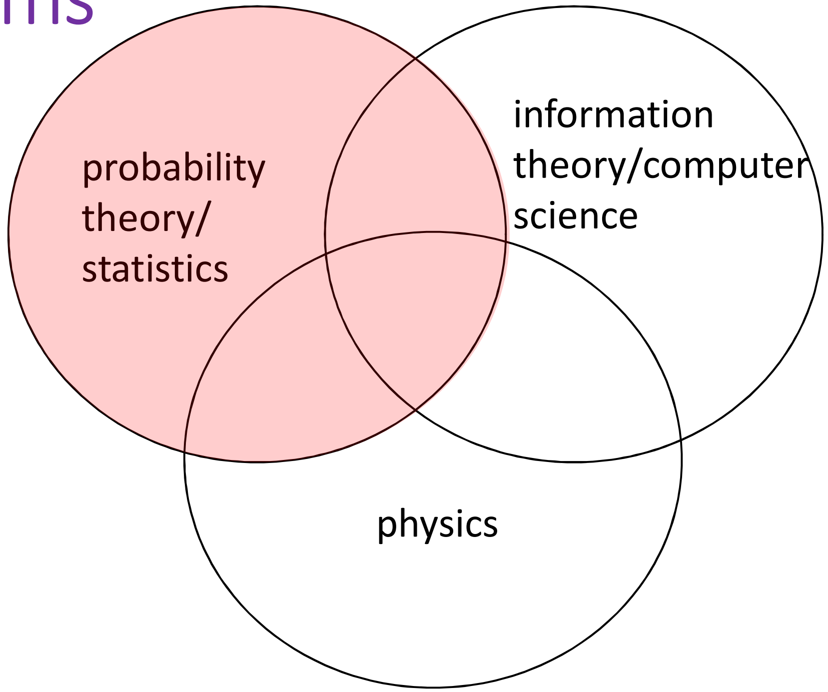




- B. de Finetti: La prévision: ses lois logiques, ses sources subjectives, 1937
- E. Stormer, Symmetric States of Infinite Tensor Products of C*-algebras, 1969
- R. L. Hudson and G. R. Moody : Locally Normal Symmetric States and an Analogue of de Finetti's Theorem, 1976
- A. Raggio and R. F. Werner: Quantum Statistical Mechanics of General Mean Field Systems, 1989
- P. Diaconis and D. Freedman: Finite Exchangeable Sequences, 1980
- C. M. Caves, C. A. Fuchs and R. Schack: Unknown Quantum States: The Quantum de Finetti Representation, 2002

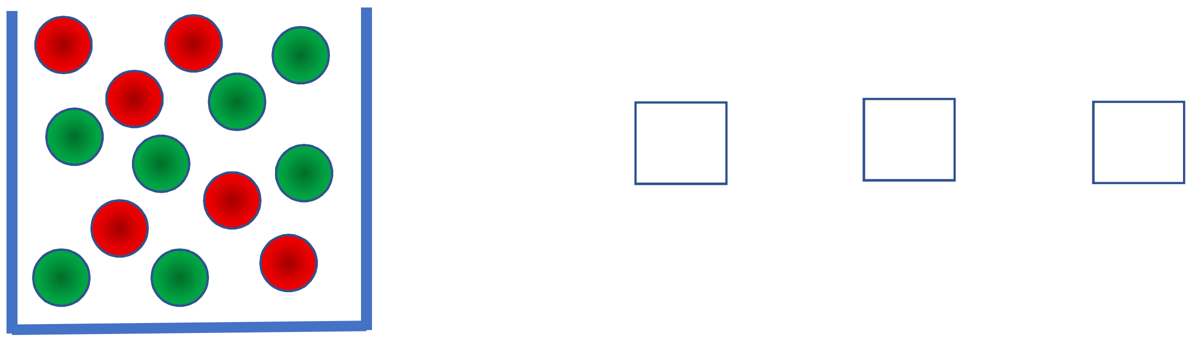
de Finetti theorems





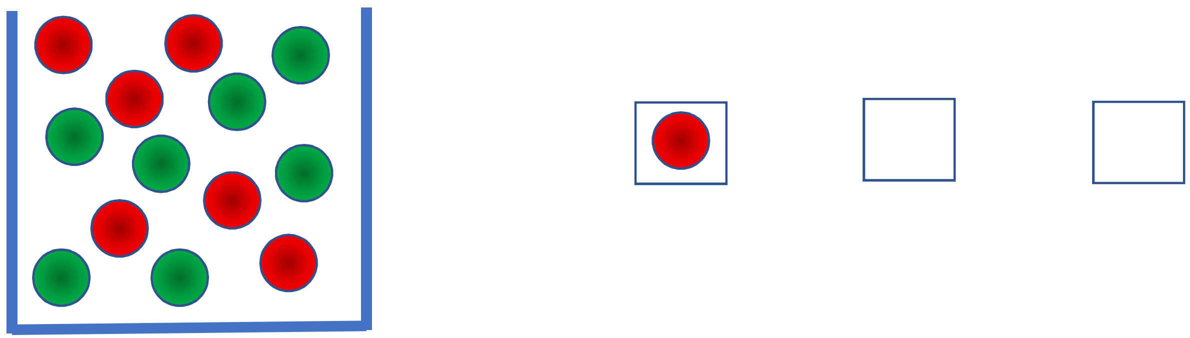
- B. de Finetti: La prévision: ses lois logiques, ses sources subjectives, 1937
- E. Stormer, Symmetric States of Infinite Tensor Products of C*-algebras, 1969
- R. L. Hudson and G. R. Moody : Locally Normal Symmetric States and an Analogue of de Finetti's Theorem, 1976
- A. Raggio and R. F. Werner: Quantum Statistical Mechanics of General Mean Field Systems, 1989
- P. Diaconis and D. Freedman: Finite Exchangeable Sequences, 1980
- C. M. Caves, C. A. Fuchs and R. Schack: Unknown Quantum States: The Quantum de Finetti Representation, 2002

Drawing k balls



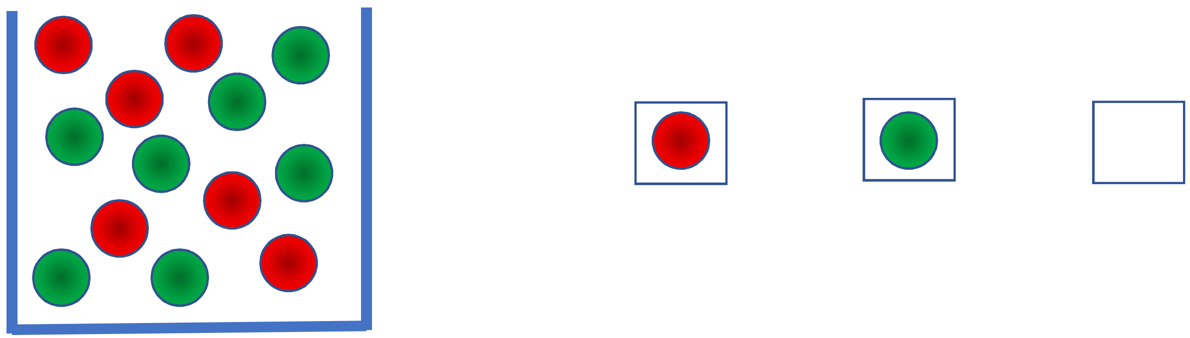
with replacement

Drawing k balls



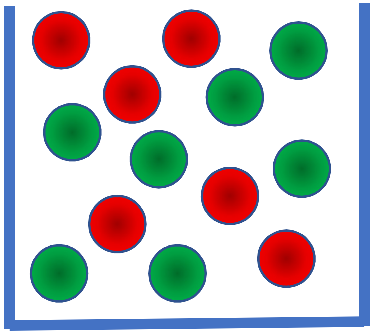
with replacement

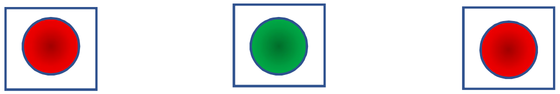
Drawing k balls



with replacement

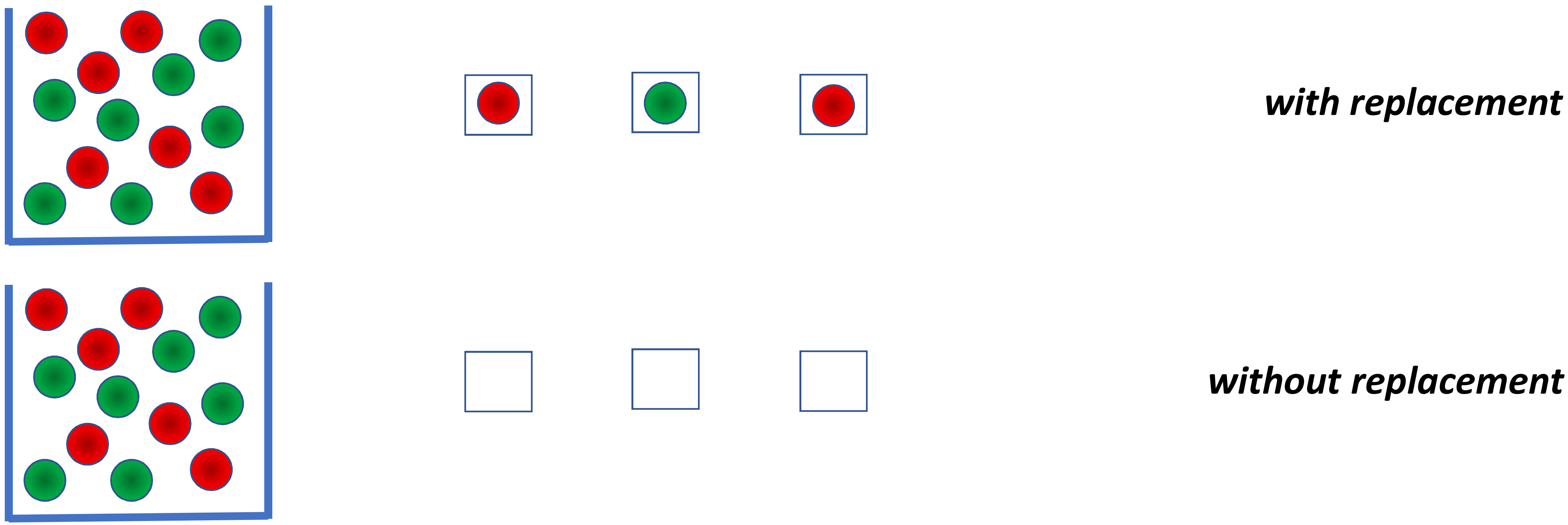
Drawing k balls





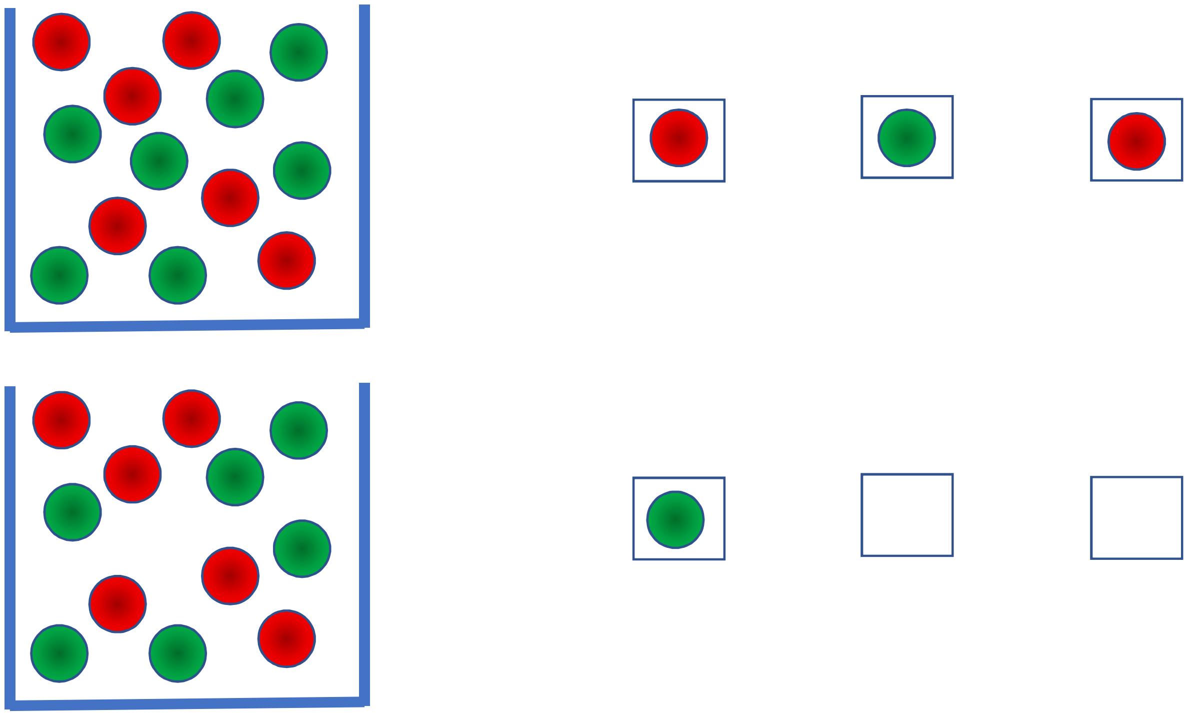
with replacement

Drawing k balls



What do these procedures have in common?

Drawing k balls

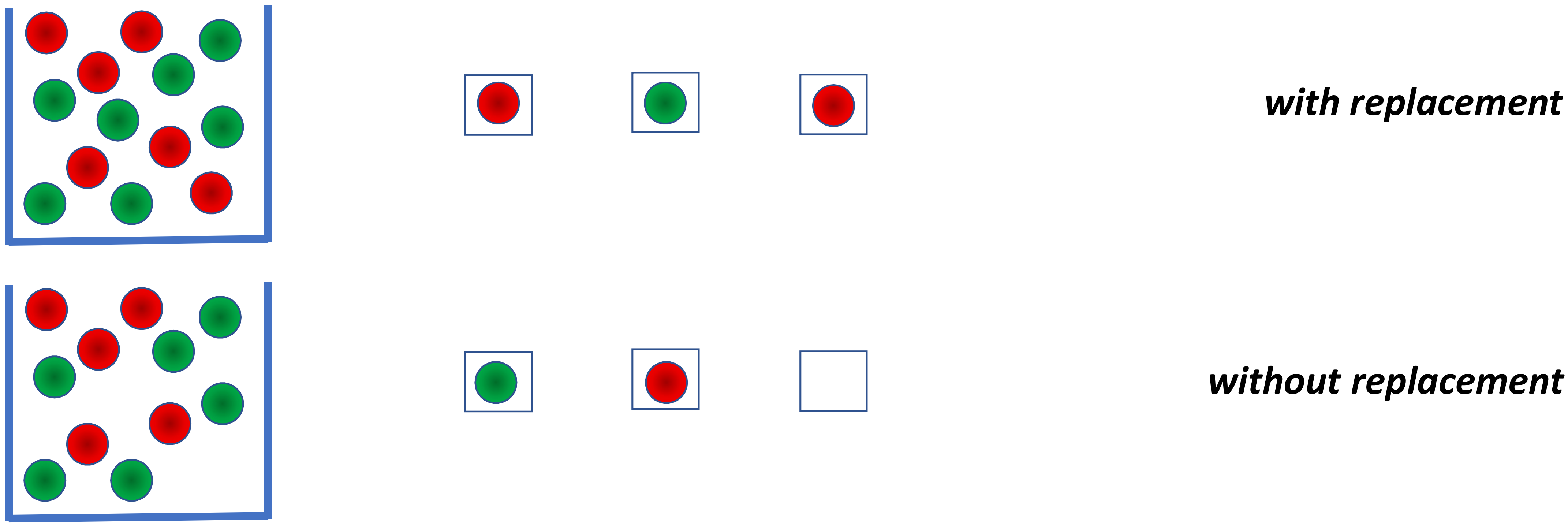


with replacement

without replacement

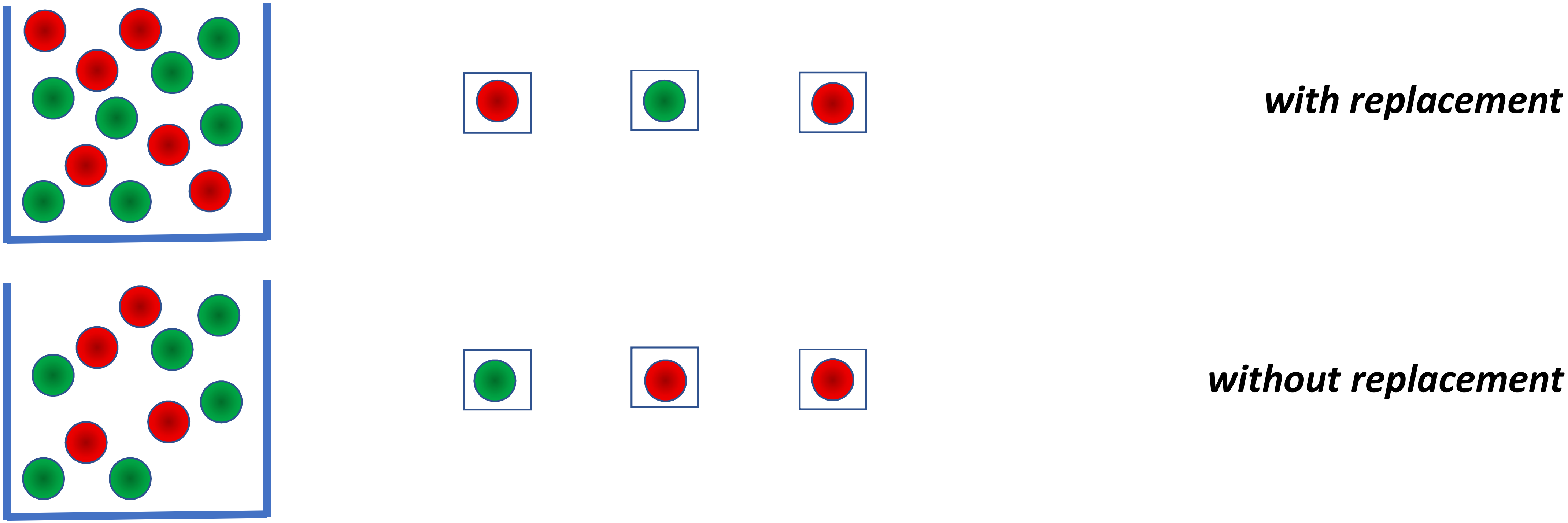
What do these procedures have in common?

Drawing k balls



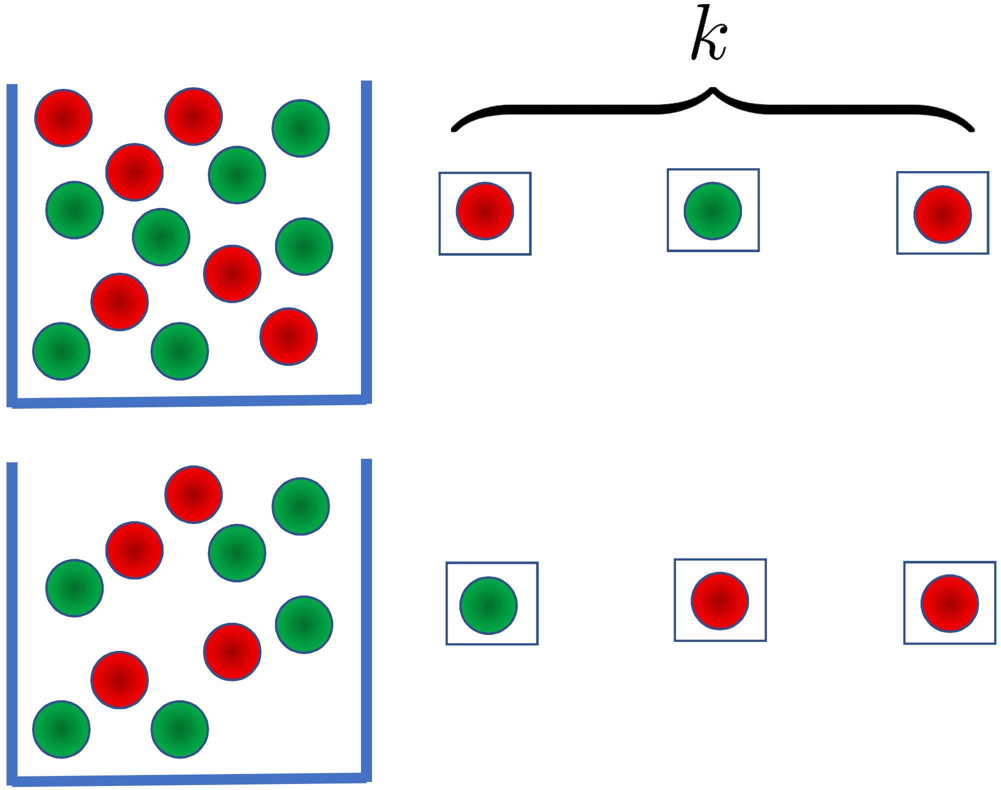
What do these procedures have in common?

Drawing k balls



What do these procedures have in common?

Drawing k of a total of n balls



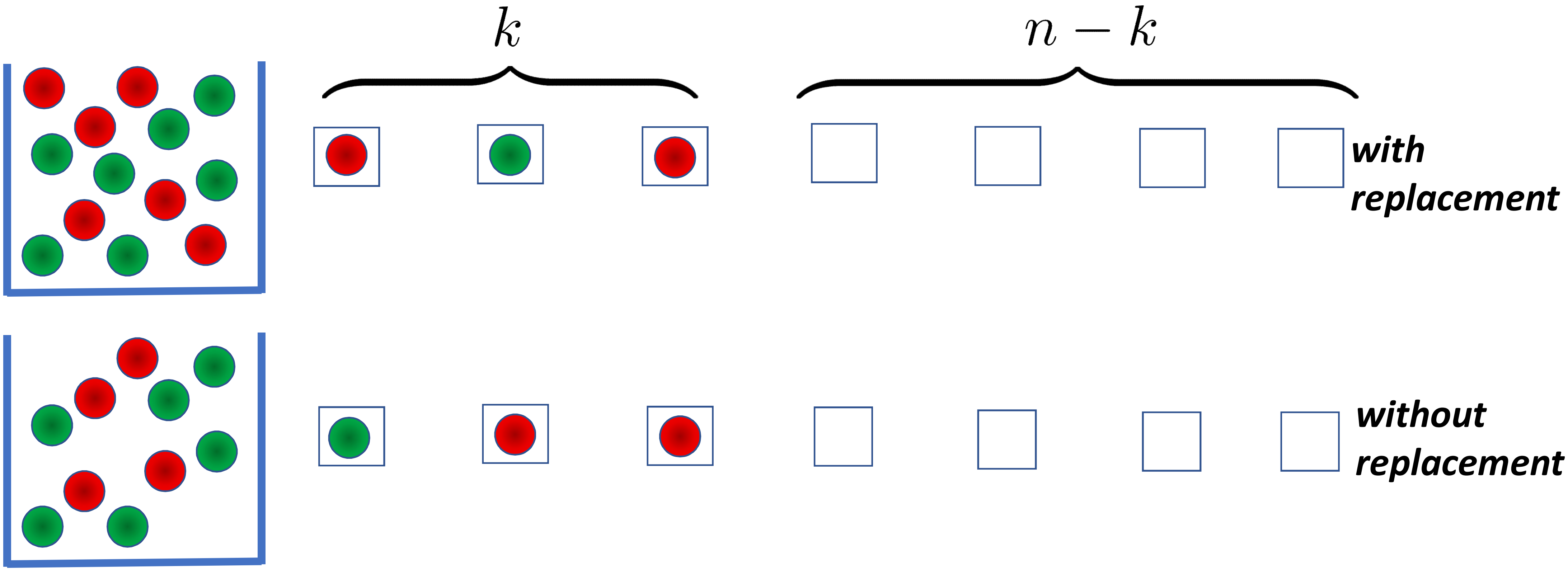
with replacement

without replacement

Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \dots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

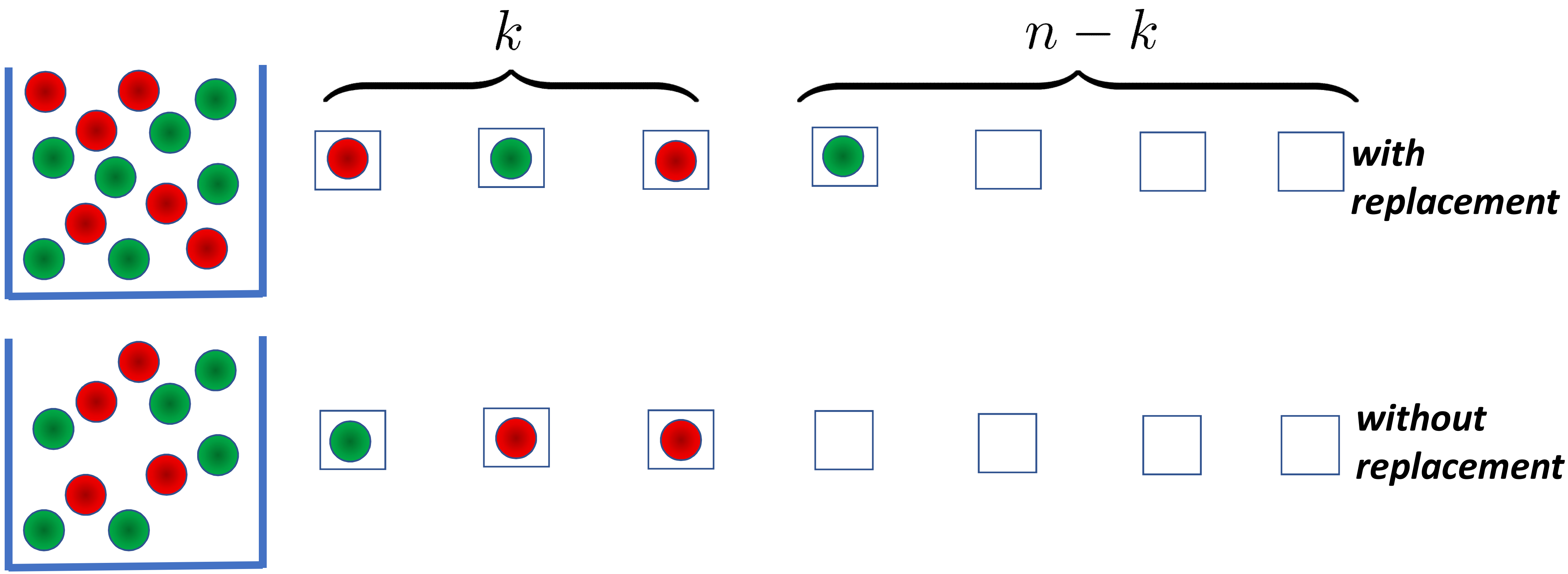
Drawing k of a total of n balls



Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \ldots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

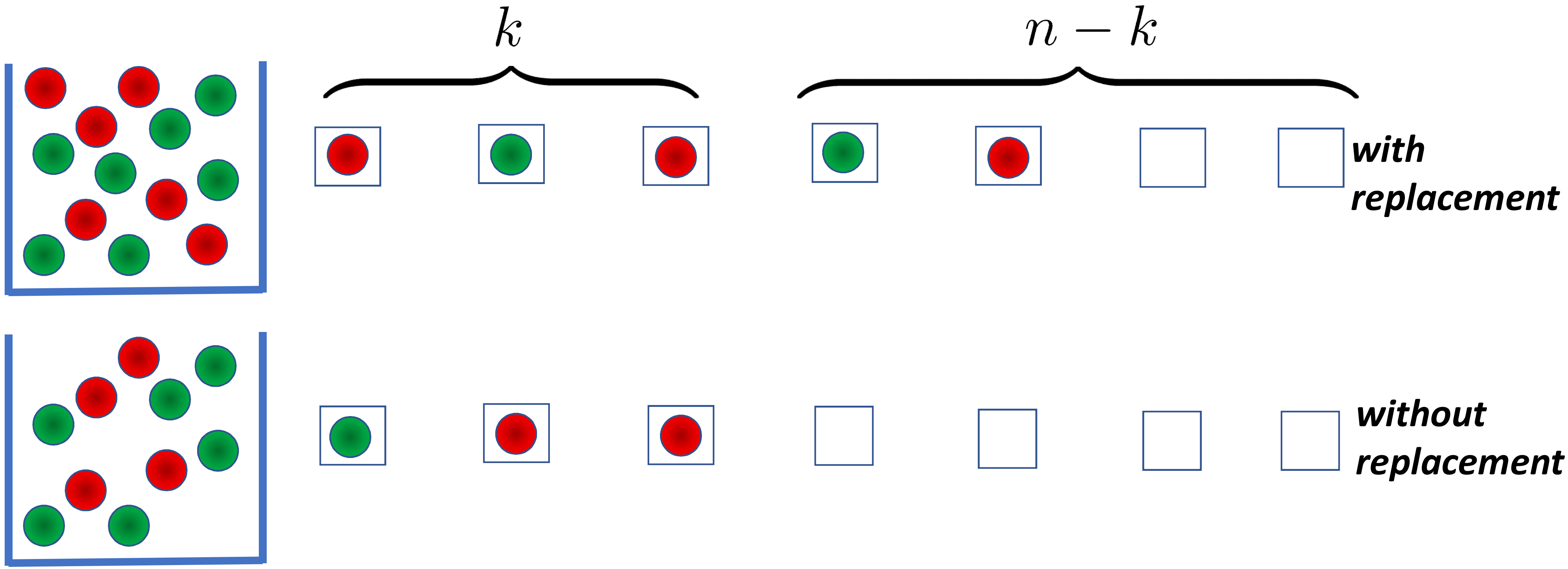
Drawing k of a total of n balls



Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \dots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

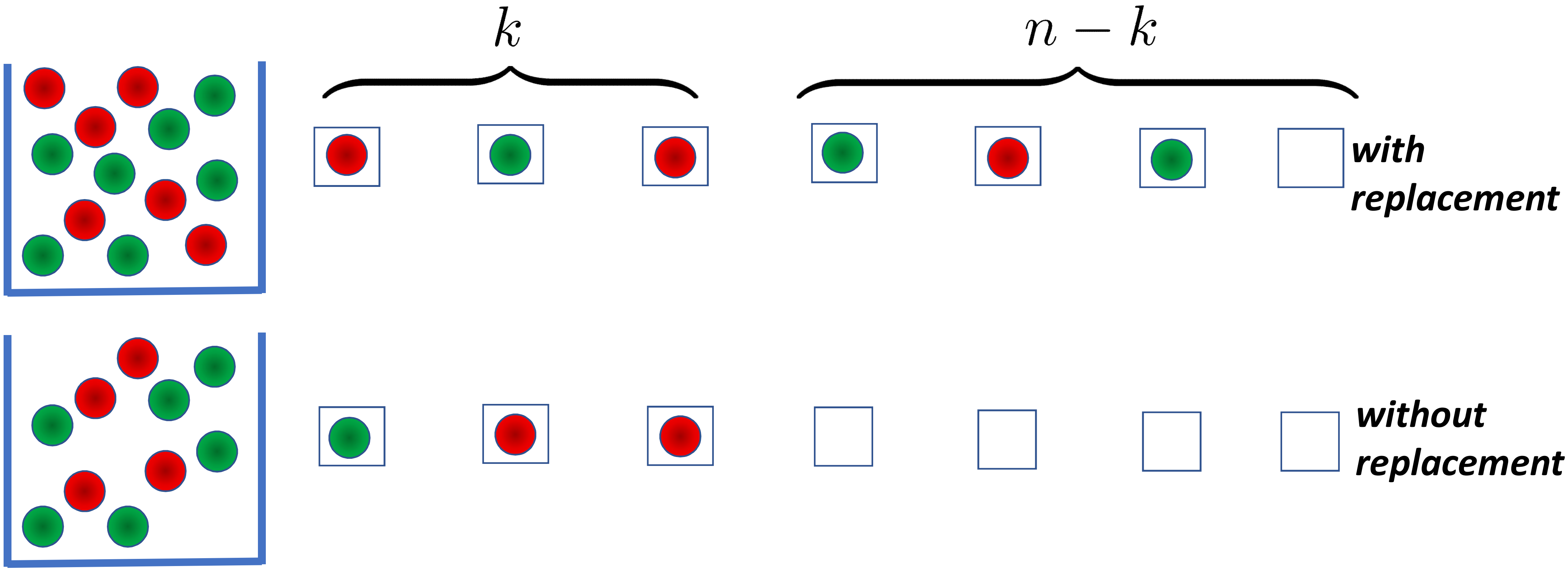
Drawing k of a total of n balls



Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \ldots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

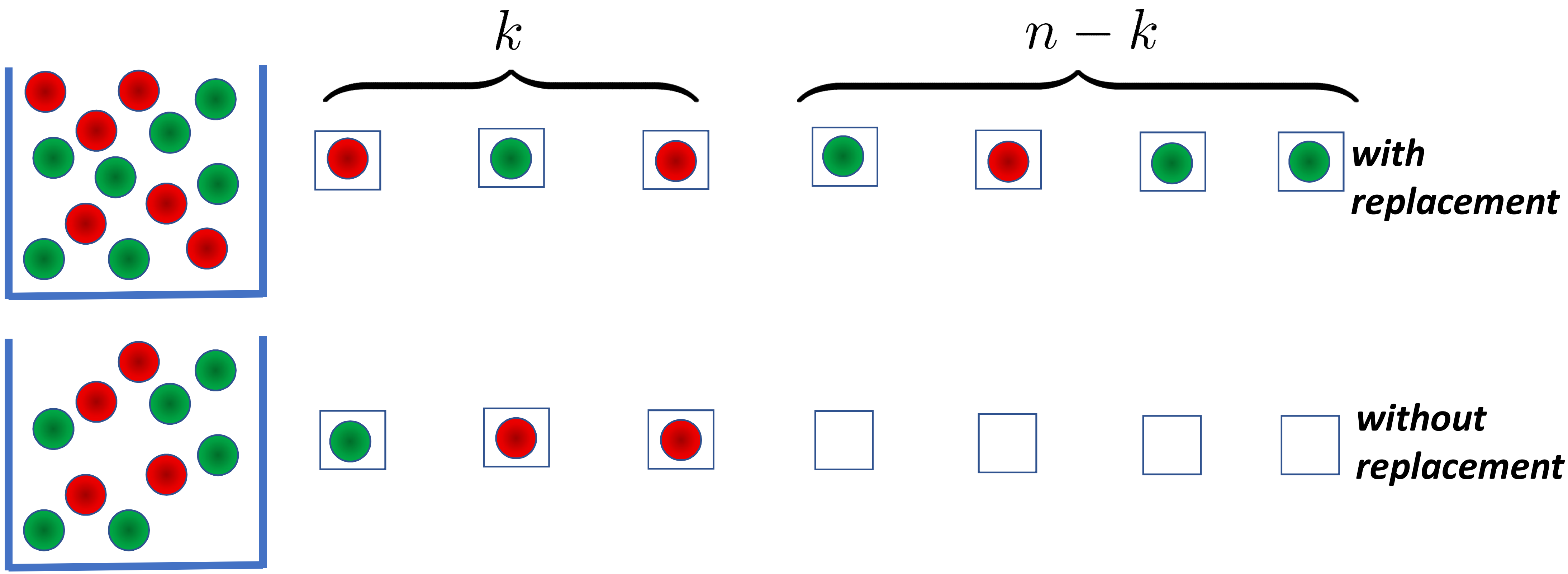
Drawing k of a total of n balls



Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \ldots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

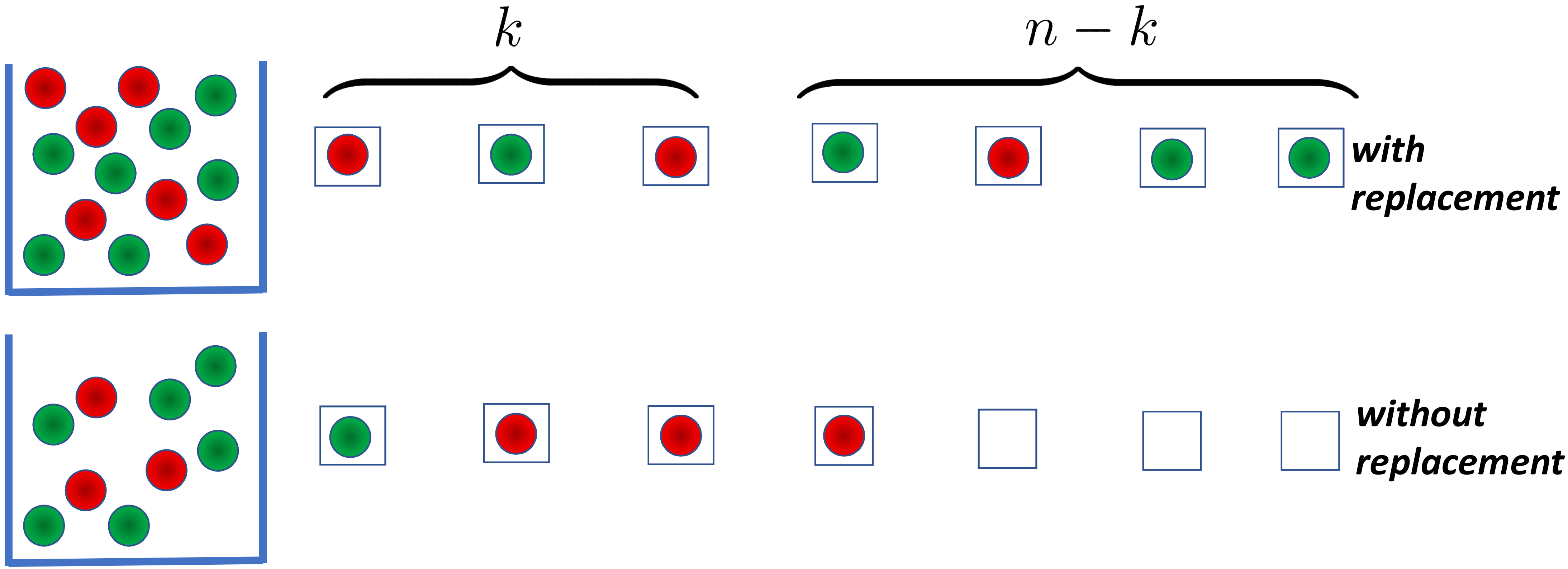
Drawing k of a total of n balls



Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \ldots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

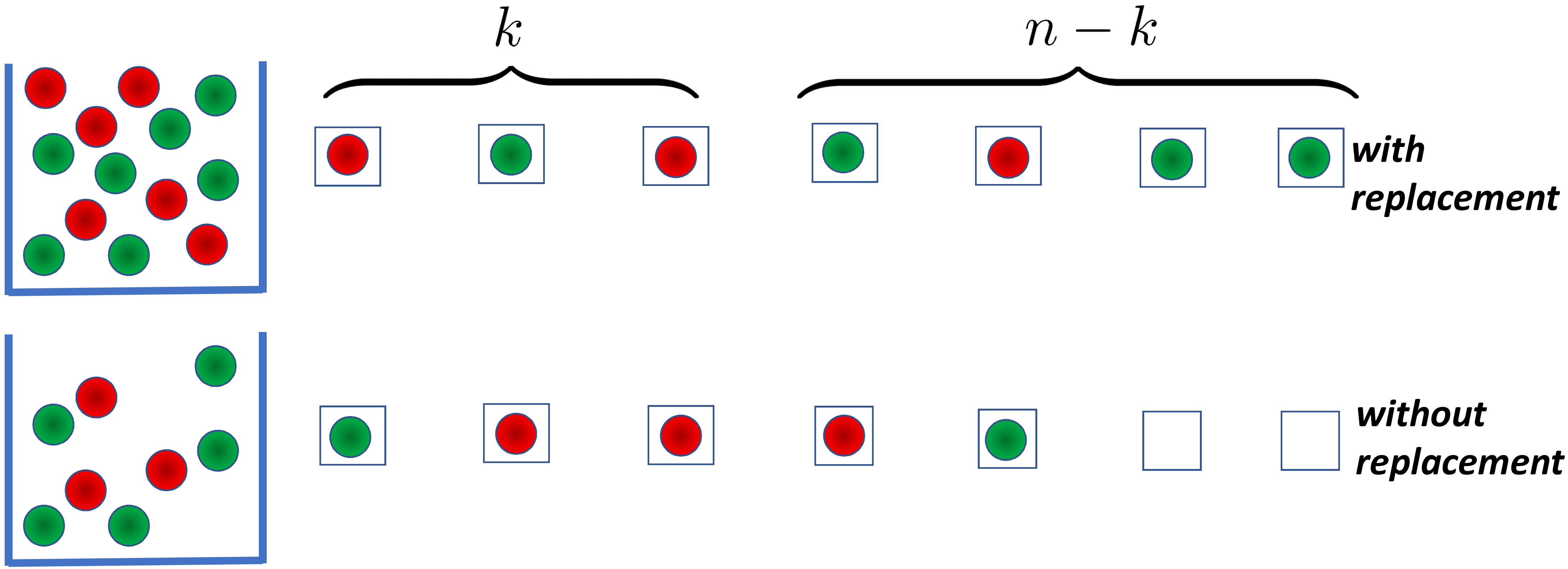
Drawing k of a total of n balls



Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \ldots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

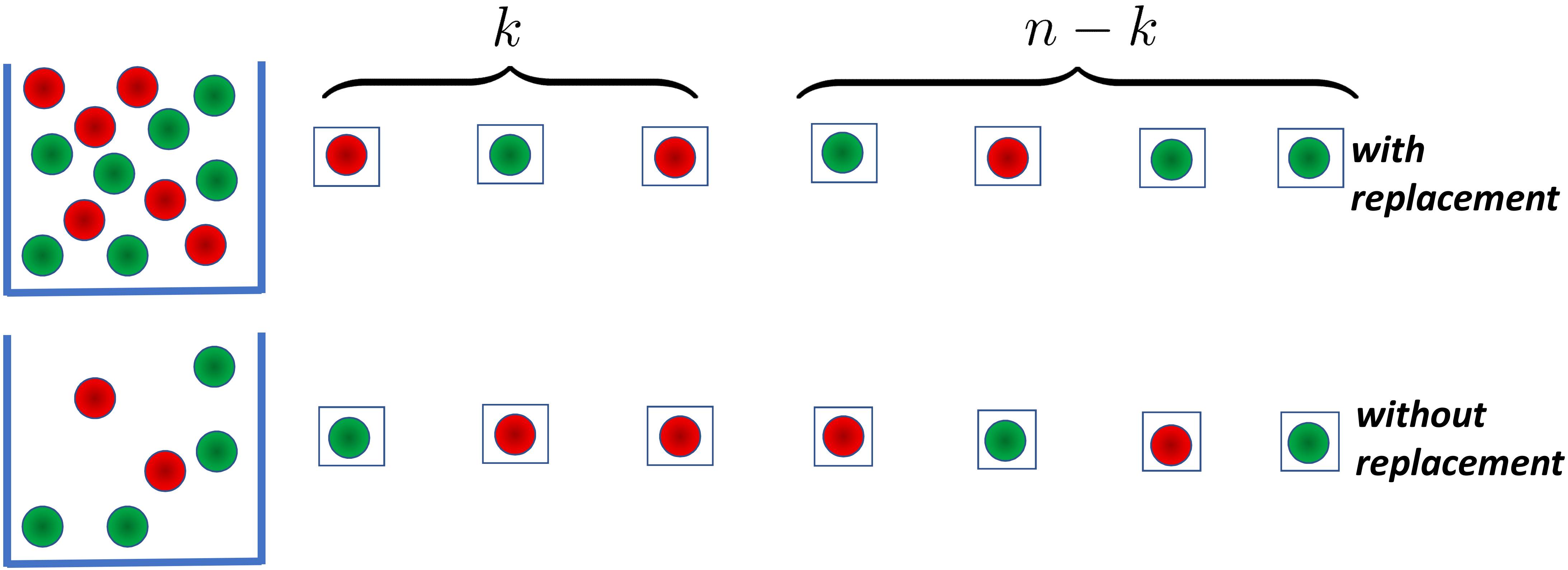
Drawing k of a total of n balls



Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \ldots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

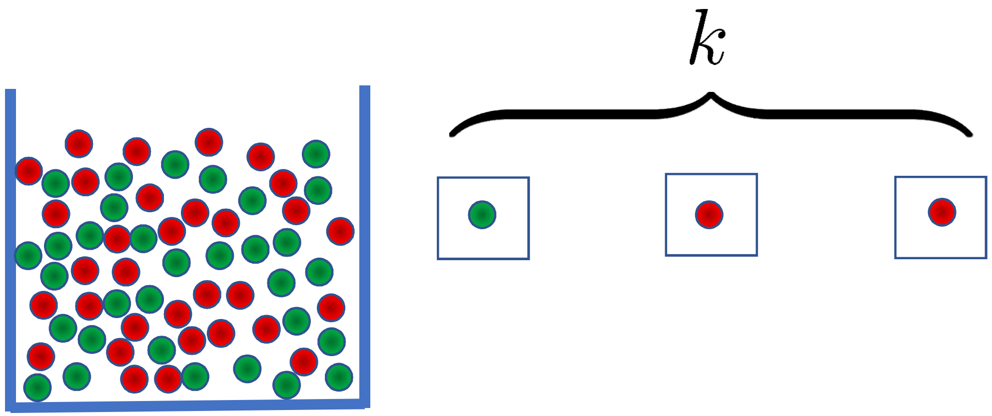
Drawing k of a total of n balls



Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \ldots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

Drawing k of *infinitely many* balls



drawing without replacement $\approx$ ***drawing with replacement***

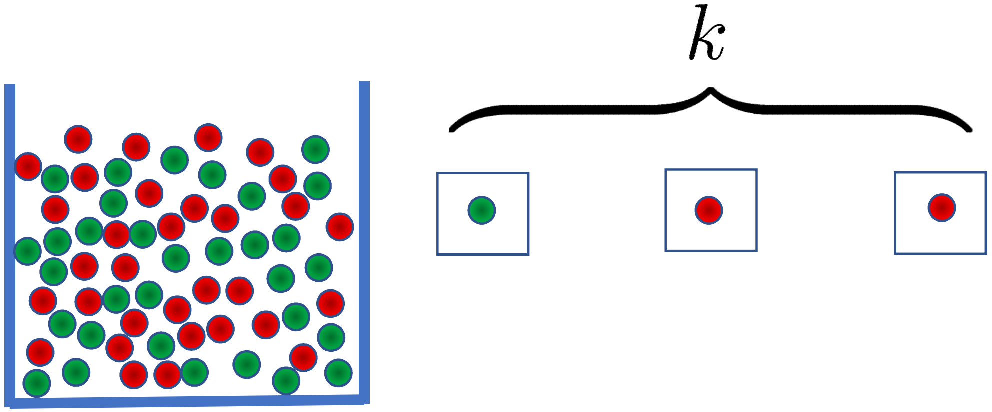
(product distribution)

De Finetti (1931): A infinitely exchangeable sequence of binary random variables is a convex combination of product distributions.

Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \ldots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

Drawing k of *many* balls



drawing without replacement $\approx$ ***drawing with replacement***

(product distribution)

Diaconis & Freedman (1980): An n-exchangeable sequence is close to a convex combination of product distributions:

$$\exists \text{ probability density function } \mu \text{ such that } \quad \left\| P_{X_1 \cdots X_k} - \int P^k d\mu(P) \right\|_1 \leq d \cdot \frac{k}{n}$$

Definition: A distribution $P_{X_1 \cdots X_k}$ of k random variables $(X_1, \ldots, X_k)$ is **n-exchangeable** if there is a distribution $P_{X_1 \cdots X_n}$ with the same marginal which is permutation-invariant, that is,

$$P_{X_{\pi(1)} \cdots X_{\pi(n)}} \equiv P_{X_1 \cdots X_n} \quad \text{for any permutation} \quad \pi \in S_n$$

From classical to quantum mechanics: more symmetry

k red or green balls, randomly chosen

probability distribution $P_{X_1 \cdots X_k}$

Vector of 2^n non-negative reals summing to 1

k qubits ("quantum colored balls")

density operator $\rho_{A_1 \cdots A_k}$

$2^n \times 2^n$ complex nonnegative matrix with trace 1

classical

quantum

From classical to quantum mechanics: more symmetry

k red or green balls, randomly chosen

probability distribution $P_{X_1 \cdots X_k}$

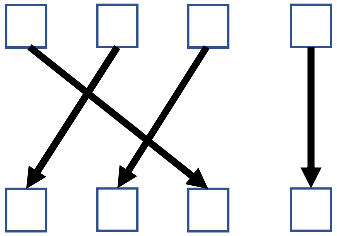
Vector of 2^n non-negative reals summing to 1

k qubits ("quantum colored balls")

density operator $\rho_{A_1 \cdots A_k}$

$2^n \times 2^n$ complex Hermitan matrix with trace 1

for any permutation $\pi \in S_k$



permuting variables

$$P_{X_1 \cdots X_k} \mapsto P_{X_{\pi(1)} \cdots X_{\pi(k)}}$$

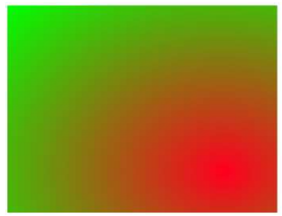
permuting subsystems

$$\rho_{A_1 \cdots A_k} \mapsto \rho_{A_{\pi(1)} \cdots A_{\pi(k)}}$$

switching colors

$\mathbb{Z}_2$

changing colors



$SU(2)$

Exchangeable multipartite quantum states

Definition: A state $\rho_{A_1 \cdots A_k}$ of k qudits is **n-exchangeable** if there is a state $\rho_{A_1 \cdots A_n}$ with the same reduced density operator

such that $\rho_{A_{\pi(1)} \cdots A_{\pi(n)}} \equiv \rho_{A_1 \cdots A_n}$ for any permutation $\pi \in S_n$

Examples:

- For any qudit state σ the state $\rho_{A_1 \cdots A_k} \equiv \sigma^{\otimes k}$ is n-exchangeable

- If μ is a probability distribution over states, then $\rho_{A_1 \cdots A_k} \equiv \int \sigma^{\otimes k} d\mu(\sigma)$ is n-exchangeable

Symmetric extension:

$$\rho_{A_1 \cdots A_n} \equiv \sigma^{\otimes n}$$

$$\rho_{A_1 \cdots A_n} \equiv \int \sigma^{\otimes n} d\mu(\sigma)$$

Exchangeable multipartite quantum states

Definition: A state $\rho_{A_1 \cdots A_k}$ of k qudits is **n-exchangeable** if there is a state $\rho_{A_1 \cdots A_n}$ with the same reduced density operator

such that $\rho_{A_{\pi(1)} \cdots A_{\pi(n)}} \equiv \rho_{A_1 \cdots A_n}$ for any permutation $\pi \in S_n$

Example:

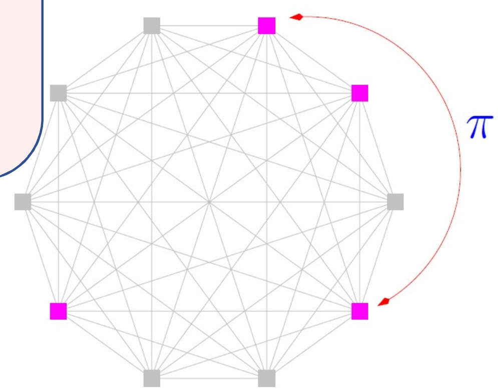
- For any permutation-invariant pure state $|\Psi\rangle \in \mathrm{Sym}^n(\mathbb{C}^d) \subset (\mathbb{C}^d)^{\otimes n}$ the reduced density operator

$$\rho_{A_1 \cdots A_k} \equiv \mathrm{tr}_{n-k} |\Psi\rangle\langle\Psi|$$

is n-exchangeable.

Symmetric extension:

$|\Psi\rangle\langle\Psi|$



Physical example: ground state of a system with pairwise (identical) interactions.

Finitely exchangeable quantum states: de Finetti theorems

Definition: A state $\rho_{A_1 \cdots A_k}$ of k qudits is **n-exchangeable** if there is a state $\rho_{A_1 \cdots A_n}$ with the same reduced density operator

such that $\rho_{A_{\pi(1)} \cdots A_{\pi(n)}} \equiv \rho_{A_1 \cdots A_n}$ for any permutation $\pi \in S_n$

Thm: An n-exchangeable quantum state ρ is close to a convex combination of product states:

$\exists$ probability measure μ such that $\left\| \rho - \int \sigma^{\otimes k} d\mu(\sigma) \right\|_1 \leq d^2 \cdot \frac{k}{n}$

- Non-commutative analog of Diaconis and Freedman's result on finitely exchangeable sequences

- Applications: separability testing, quantum key distribution, variational physics

One-and-a-half quantum de Finetti theorems

Matthias Christandl,* Robert König,† Graeme Mitchison,‡ and Renato Renner§
Centre for Quantum Computation, DAMTP, University of Cambridge, Cambridge CB3 0WA, UK
(Dated: October 4, 2008)

de Finetti theorems for unitarily invariant states

One-and-a-half quantum de Finetti theorems

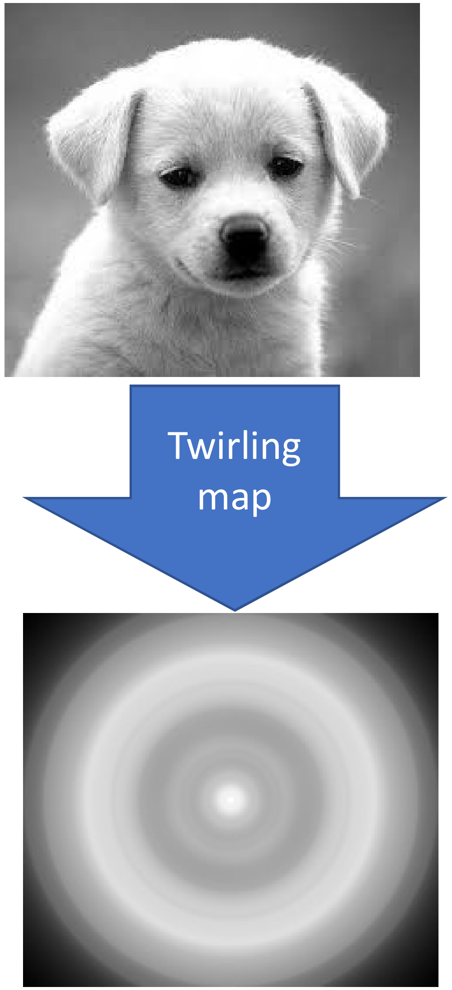
Matthias Christandl,* Robert König,† Graeme Mitchison,‡ and Renato Renner§
Centre for Quantum Computation, DAMTP, University of Cambridge, Cambridge CB3 0WA, UK
(Dated: October 4, 2008)

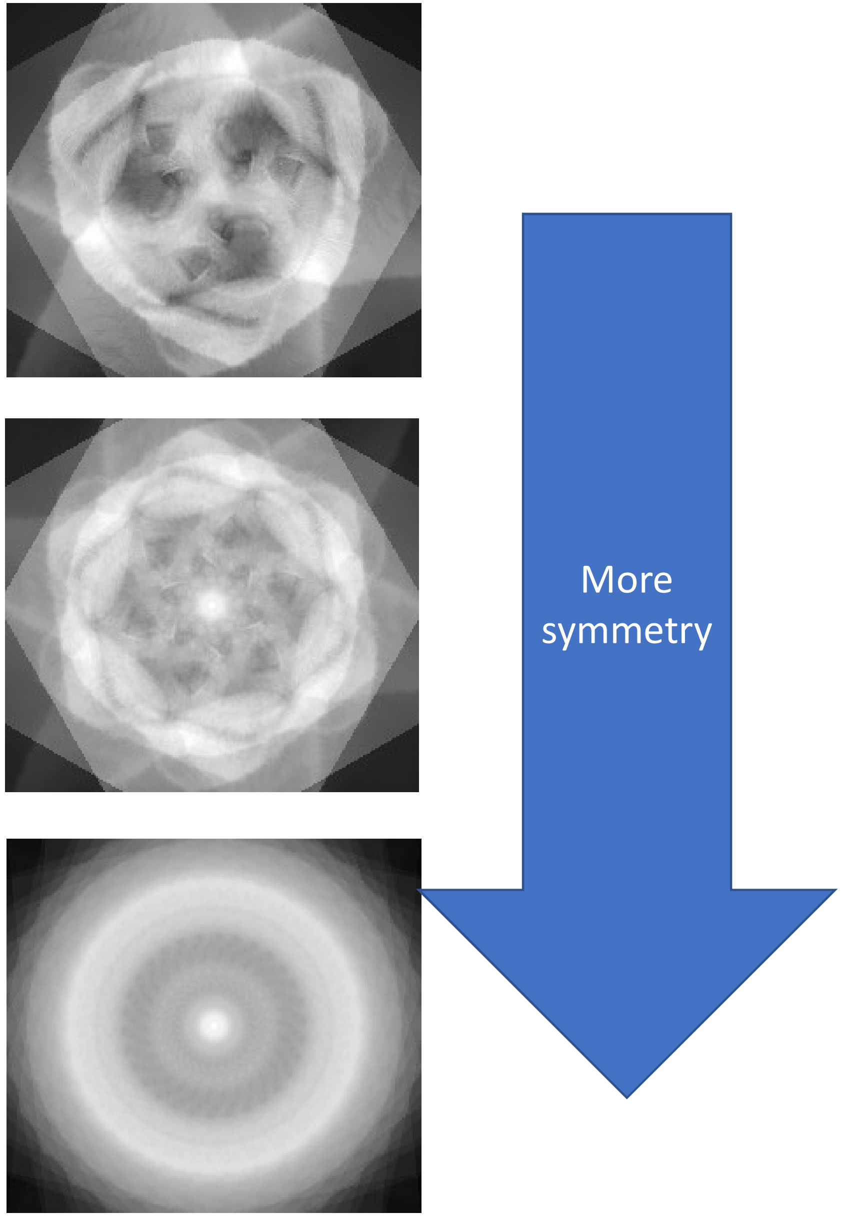
A dual de Finetti theorem

Graeme Mitchison*
Centre for Quantum Computation, DAMTP, University of Cambridge, Cambridge CB3 0WA, UK

The quantum de Finetti theorem says that, given a symmetric state, the state obtained by tracing out some of its subsystems approximates a convex sum of power states. The more subsystems are traced out, the better this approximation becomes. Schur-Weyl duality suggests that there ought to be a dual result that applies to a unitarily invariant state rather than a symmetric state. Instead of tracing out a number of subsystems, one traces out part of every subsystem. The theorem then asserts that the resulting state approximates the fully mixed state, and the larger the dimension of the traced-out part of each subsystem, the better this approximation becomes. This paper gives a number of propositions together with their dual versions, to show how far the duality holds.

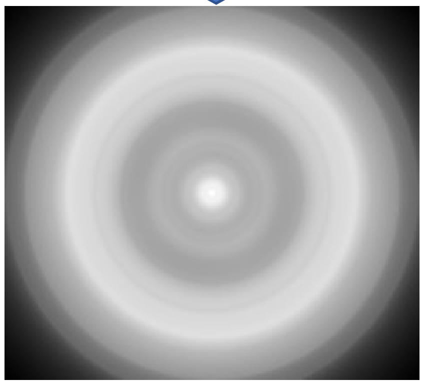
increasing symmetry: the twirling map



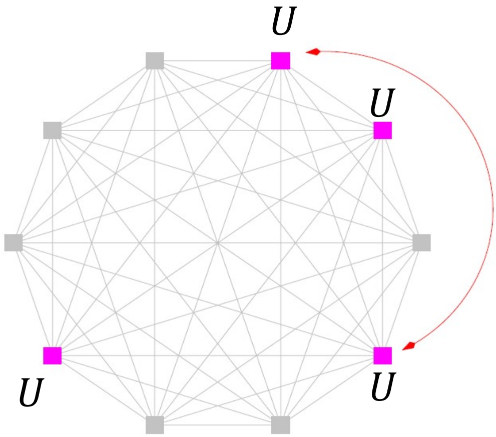


increasing symmetry:
the twirling map

Twirling map



1. Take any n-exchangeable state $\sigma = \sigma_{A_1 \cdots A_k}$



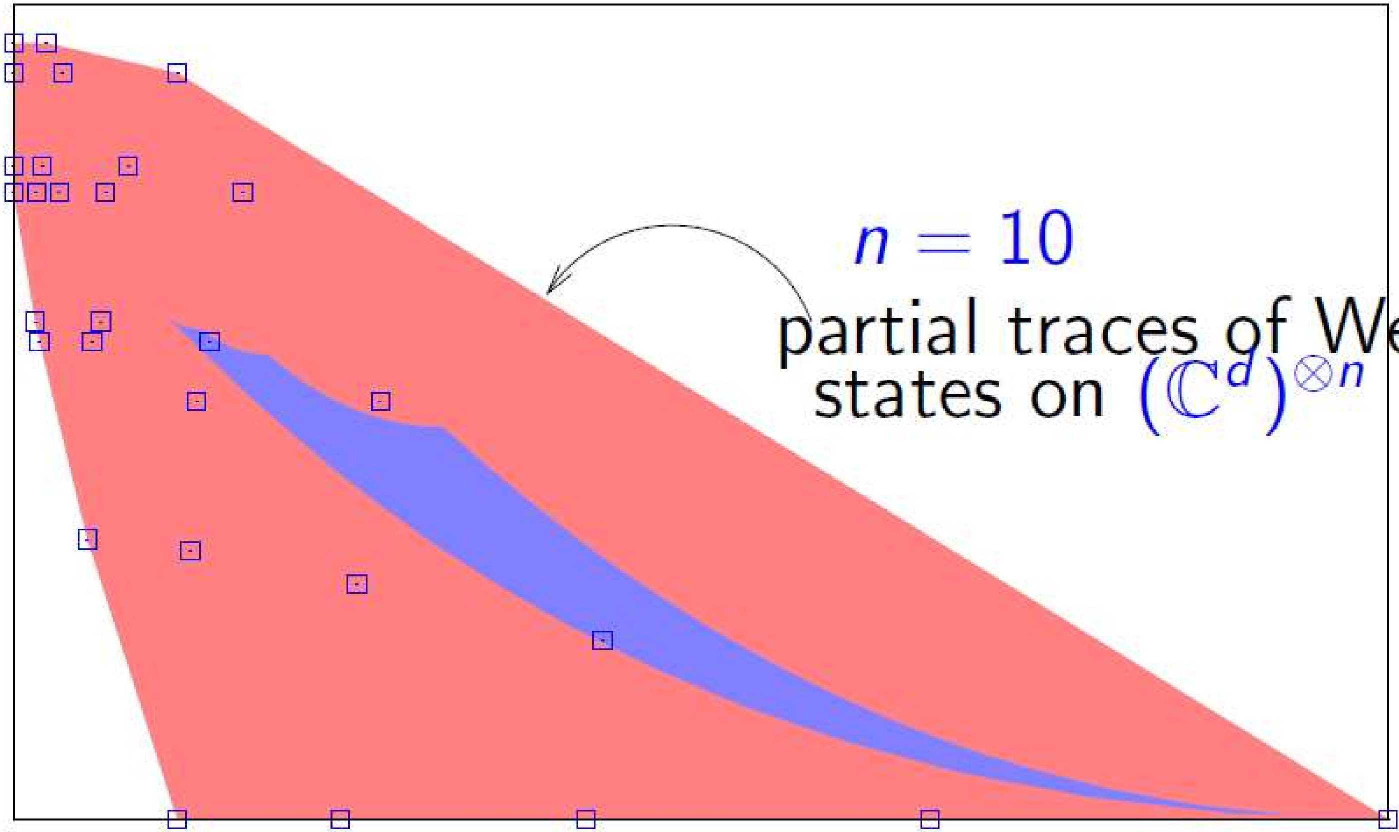
2. Apply a (Haar-random unitary) rotation U to each subsystem.

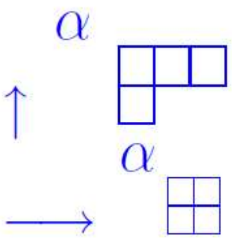
The `twirled' state $\mathbb{T}(\sigma) = \int U^{\otimes k} \sigma (U^*)^{\otimes k} dU$

is n-exchangeable and has the additional symmetry

$$U^{\otimes k} \mathbb{T}(\sigma) (U^*)^{\otimes k} = \mathbb{T}(\sigma)$$ **"unitary invariance"**

n-exchangeable unitarily invariant states

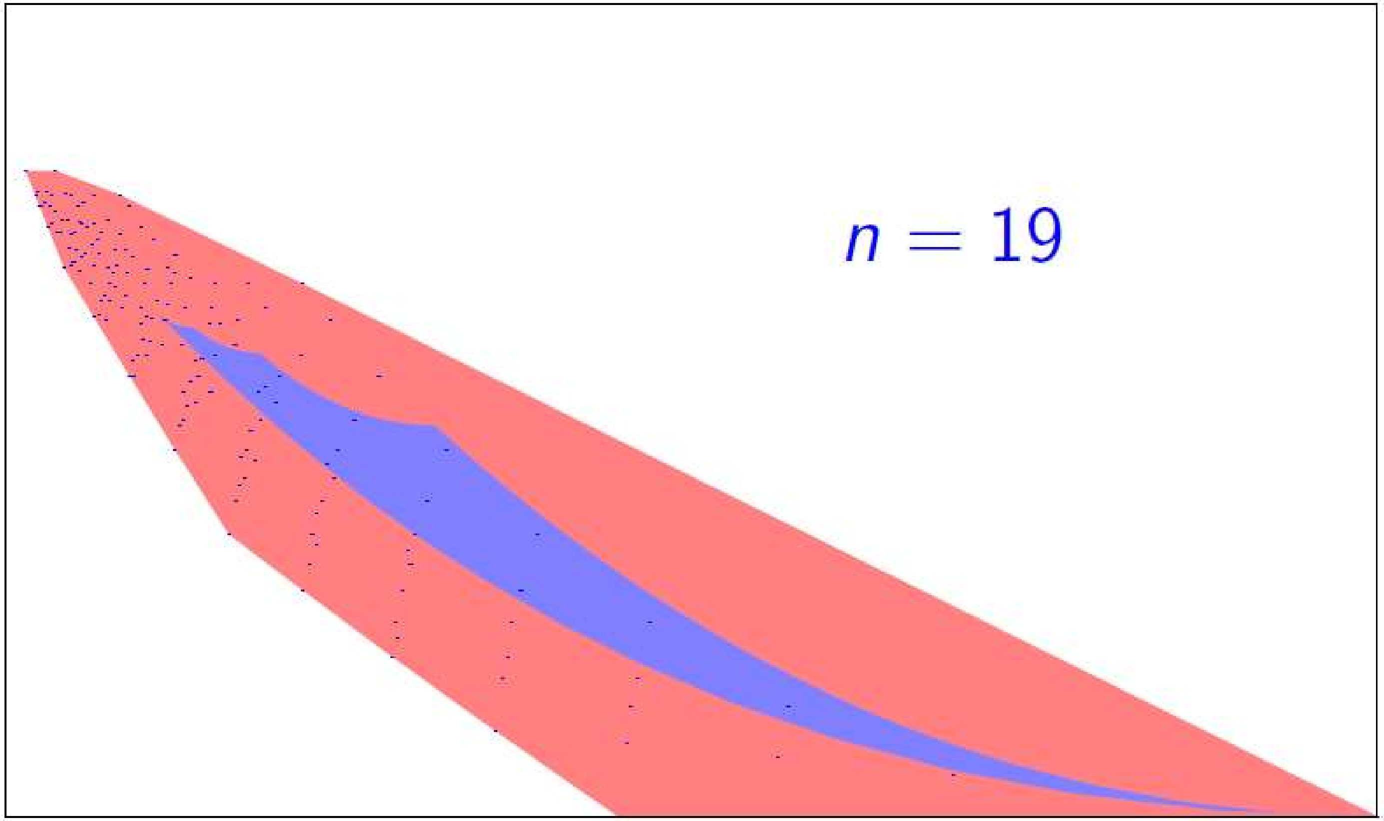


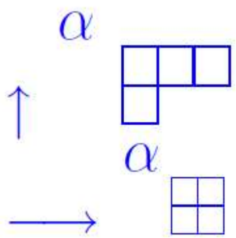


form a **polytope**.

Extreme points are partial traces of (normalized) Young projectors.

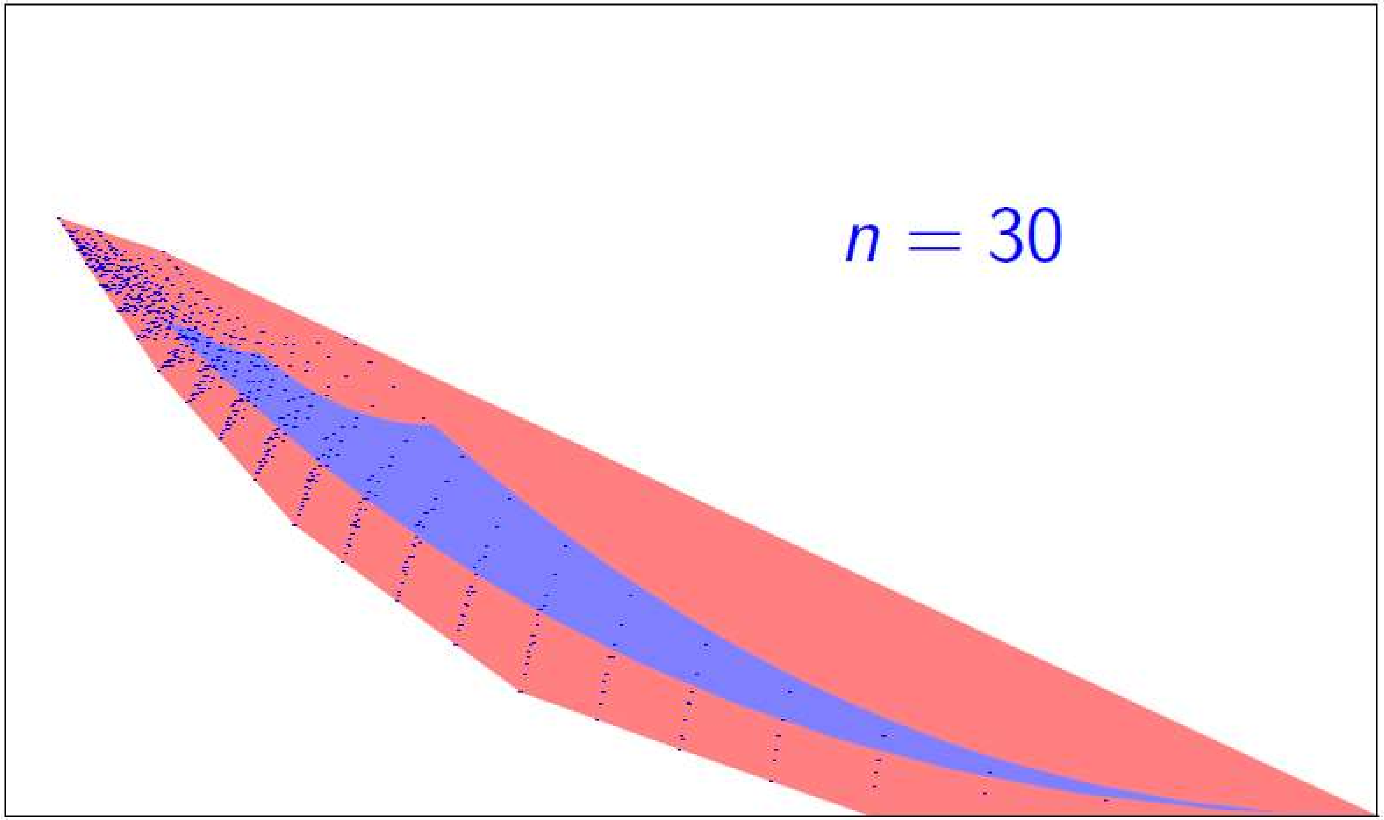
n-exchangeable unitarily invariant states

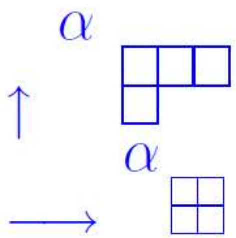




form a **polytope**.
Extreme points are partial traces of (normalized) Young projectors.

n-exchangeable unitarily invariant states

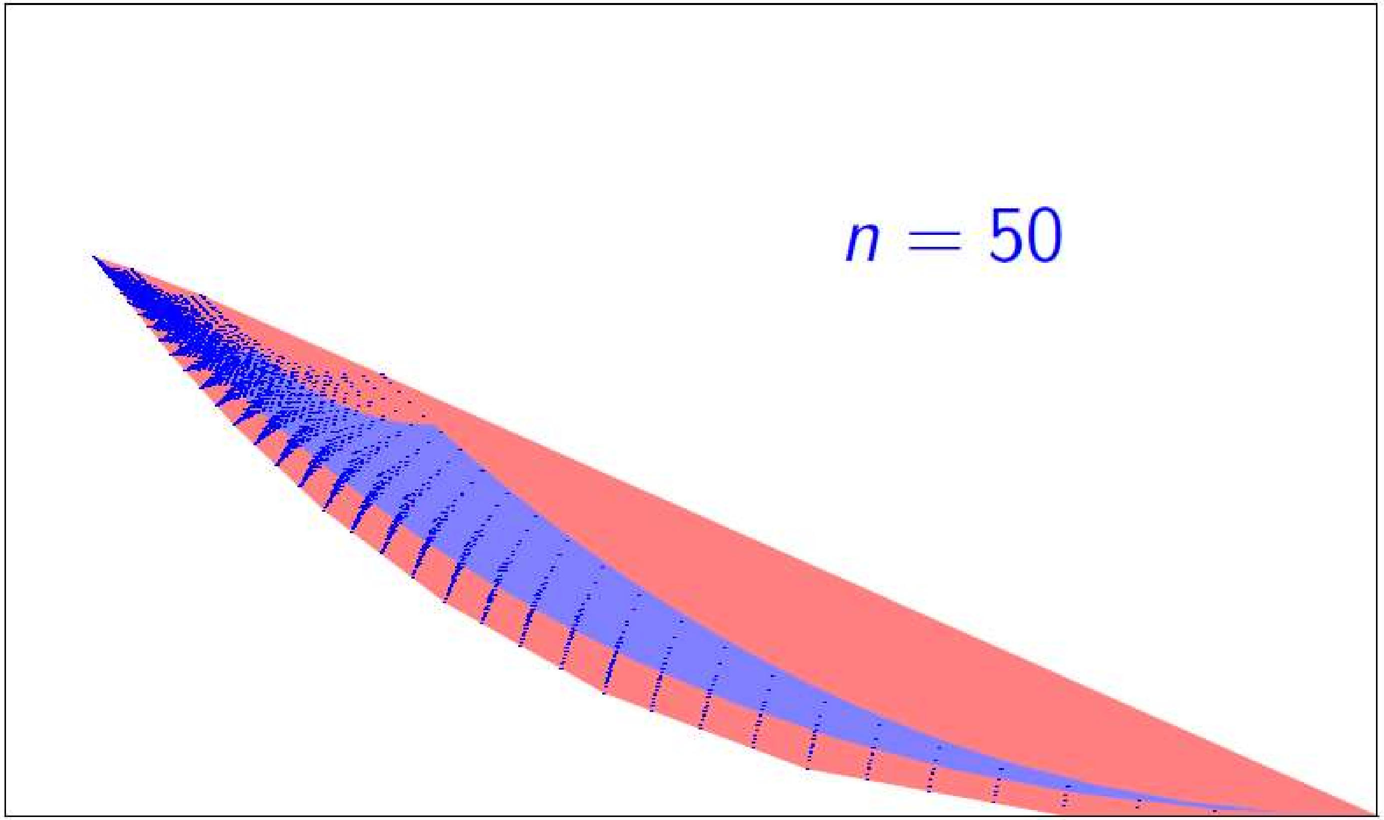


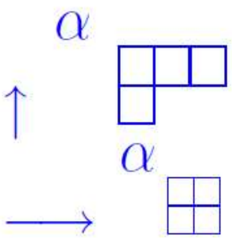


form a **polytope**.

Extreme points are partial traces of (normalized) Young projectors.

n-exchangeable unitarily invariant states

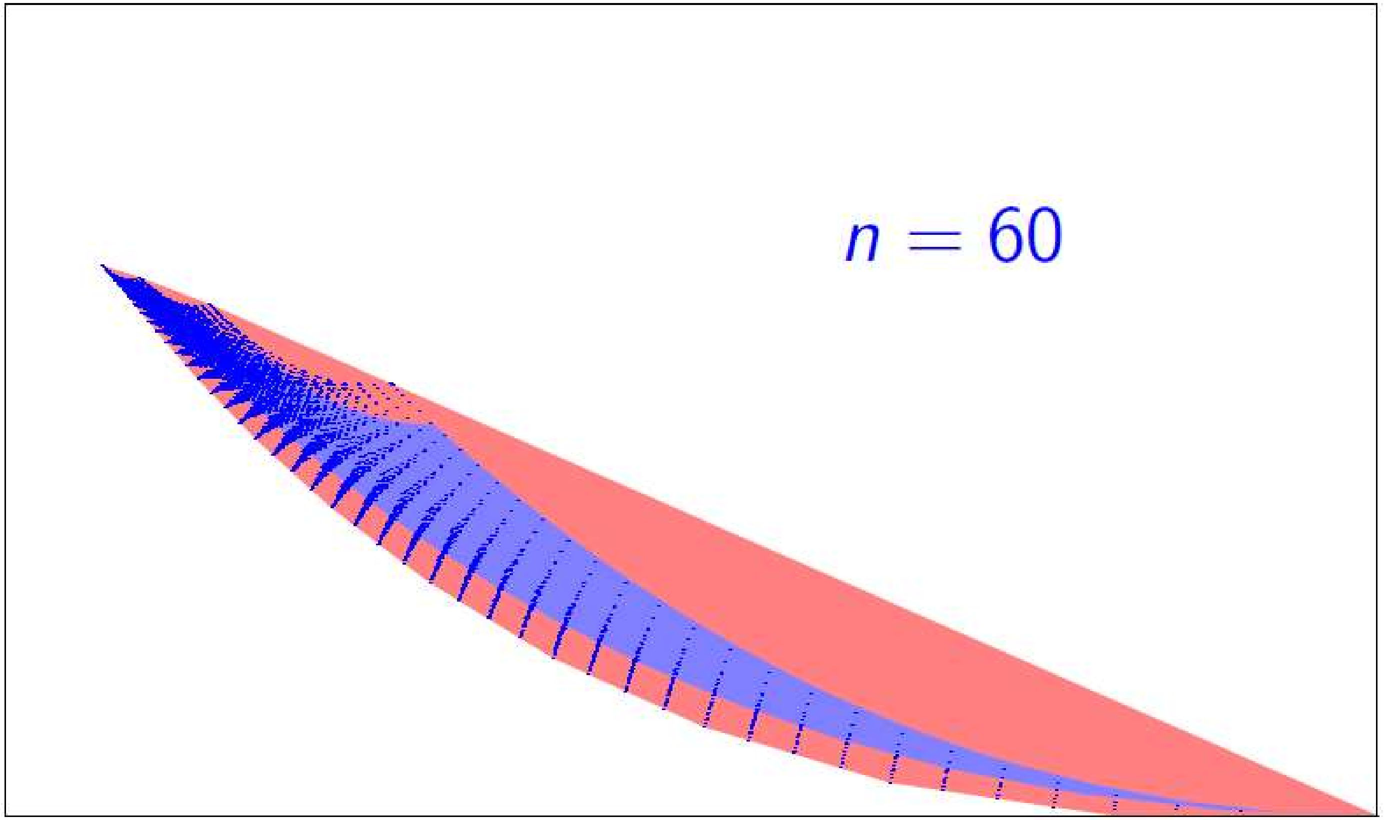


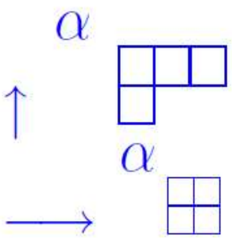


form a **polytope**.

Extreme points are partial traces of (normalized) Young projectors.

n-exchangeable unitarily invariant states

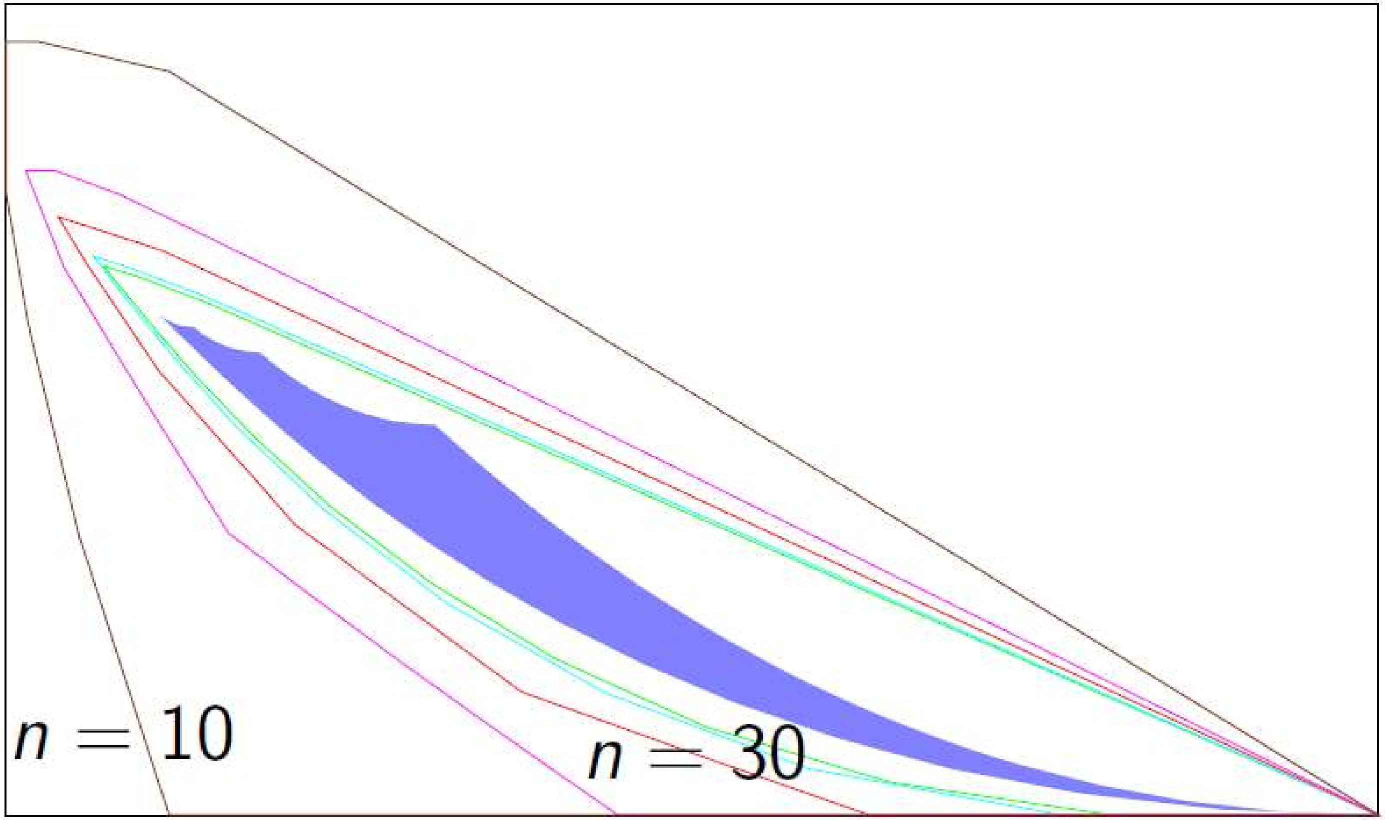


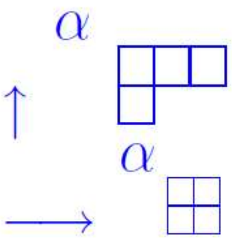


form a **polytope**.

Extreme points are partial traces of (normalized) Young projectors.

n-exchangeable unitarily invariant states





form a **polytope**.

Extreme points are partial traces of (normalized) Young projectors.

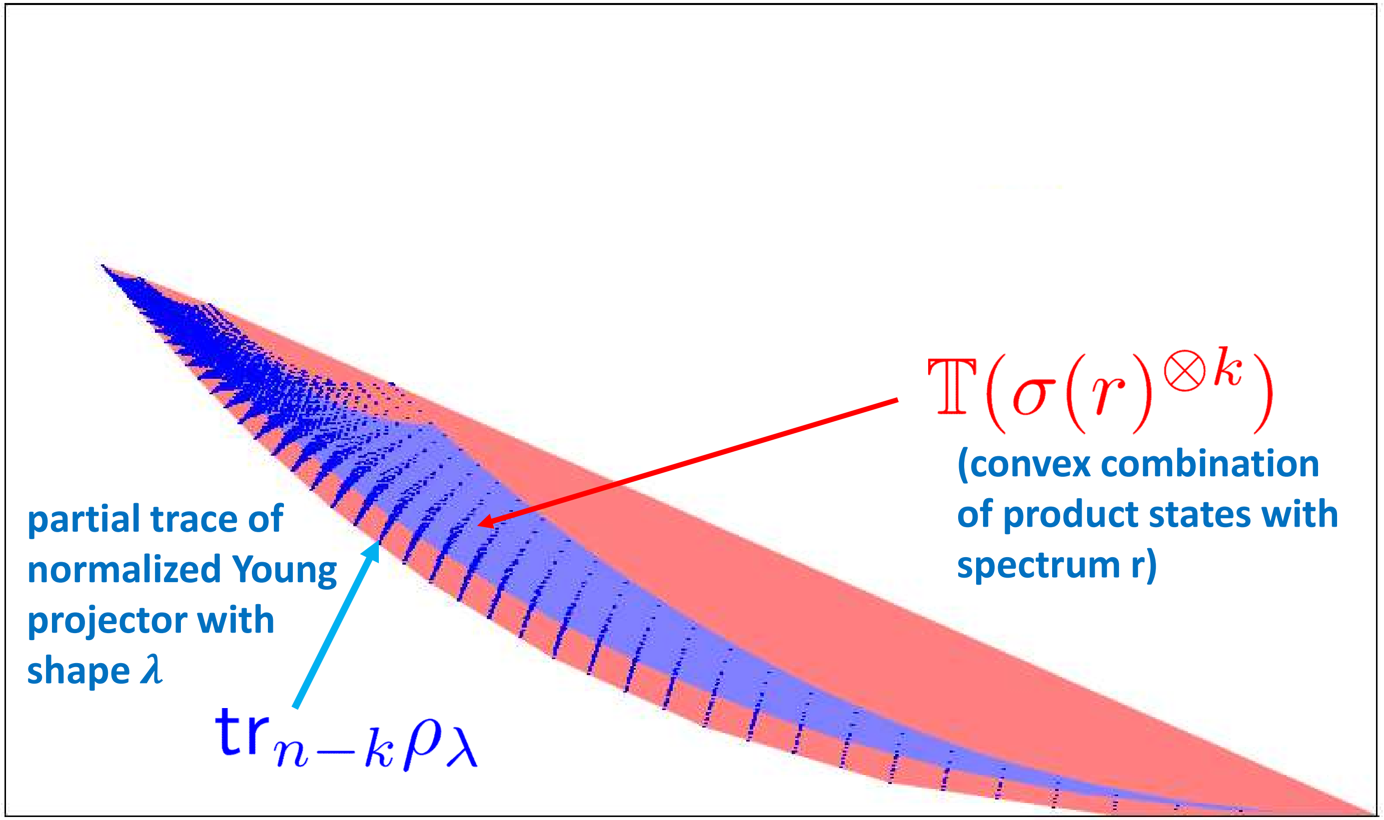
n-exchangeable unitarily invariant states

The ½ de Finetti theorem bounds the distance of the polytope(s) to the convex hull of the blue set.

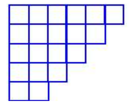
Schur-Weyl duality

$(\mathbb{C}^d)^{\otimes n} \cong \bigoplus_\lambda U_\lambda \otimes V_\lambda$

ρ_λ the normalized projector onto $U_\lambda \otimes V_\lambda$



Young diagram

$\lambda =$ 

$\downarrow$

$r = (\lambda_1/n, \ldots, \lambda_d/n)$

spectrum

This choice yields a de Finetti theorem!

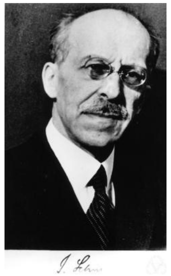
Key lemma: $\left\|\mathrm{tr}_{n-k}\rho_\lambda - \mathbb{T}(\sigma(r)^{\otimes k})\right\| = \frac{1}{2}\sum_{\mu \in \mathrm{Par}(k,d)} \dim V_\mu \cdot \left|\frac{s^\star_\mu(\lambda)}{(n\downarrow k)} - s_\mu(r)\right|$

Old and new mathematics combined

Schur function

$$s_\mu(\lambda_1,\dots,\lambda_d) = \sum_T \prod_{\alpha\in\mu} \lambda_{T(\alpha)}$$

T semistandard Young tableaux
α box



Issai Schur, 1875-1941

shifted Schur function

$$s^*_\mu(\lambda_1,\dots,\lambda_d) = \sum_T \prod_{\alpha\in\mu} (\lambda_{T(\alpha)} - c(\alpha))$$

$$c((i,j)) = j - i$$

SHIFTED SCHUR FUNCTIONS

Andrei Okounkov[1] and Grigori Olshanski

Institute for Problems of Information Transmission
Bolshoy Karetny 19, 101447 Moscow GSP–4, Russia

E-mail: okounkov@ippi.ac.msk.su, olsh@ippi.ac.msk.su

Abstract

The classical algebra Λ of symmetric functions has a remarkable deformation Λ^*, which we call the algebra of shifted symmetric functions. In the latter algebra, there is a distinguished basis formed by shifted Schur functions s^*_μ, where μ ranges over the set of all partitions. The main significance of the shifted Schur functions is that they determine a natural basis in $Z(\mathfrak{gl}(n))$, the center of the universal enveloping algebra $U(\mathfrak{gl}(n))$, $n = 1, 2, \dots$.

The functions s^*_μ are closely related to the factorial Schur functions introduced by Biedenharn and Louck and further studied by Macdonald and other authors.

A part of our results about the functions s^*_μ has natural classical analogues (com-
Jacobi–Trudi identity, Pieri formula).
ction with the binomial formula for
or the dimension of skew shapes λ/μ,
f the functions s^*_μ by their vanishing
metrization map $S(\mathfrak{gl}(n)) \to U(\mathfrak{gl}(n))$.

for the unitary groups $U(n)$ and symmetric groups $S(n)$ as $n \to \infty$.

Key lemma: $\left\|\mathrm{tr}_{n-k}\rho_\lambda - \mathbb{T}(\sigma(r)^{\otimes k})\right\| = \frac{1}{2}\sum_{\mu\in\mathrm{Par}(k,d)} \dim V_\mu \cdot \left|\frac{s^\star_\mu(\lambda)}{(n\downarrow k)} - s_\mu(r)\right|$

Old and new mathematics combined

Schur function

$$s_\mu(\lambda_1,\ldots,\lambda_d) = \sum_T \prod_{\alpha\in\mu} \lambda_{T(\alpha)}$$

shifted Schur function

$$s^*_\mu(\lambda_1,\ldots,\lambda_d) = \sum_T \prod_{\alpha\in\mu} (\lambda_{T(\alpha)} - c(\alpha))$$

$\begin{array}{\|c\|c\|}\hline 1 & 1 \\ \hline 2 \\ \cline{1-1}\end{array}$	$\begin{array}{\|c\|c\|}\hline 1 & 1 \\ \hline 3 \\ \cline{1-1}\end{array}$	$\begin{array}{\|c\|c\|}\hline 1 & 2 \\ \hline 2 \\ \cline{1-1}\end{array}$	$\begin{array}{\|c\|c\|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}$	$\begin{array}{\|c\|c\|}\hline 1 & 3 \\ \hline 3 \\ \cline{1-1}\end{array}$	$\begin{array}{\|c\|c\|}\hline 2 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}$	$\begin{array}{\|c\|c\|}\hline 2 & 3 \\ \hline 3 \\ \cline{1-1}\end{array}$
$\lambda_1^2\lambda_2$	$\lambda_1^2\lambda_3$	$\lambda_1\lambda_2^2$	$\lambda_1\lambda_2\lambda_3$	$\lambda_1\lambda_3^2$	$\lambda_2^2\lambda_3$	$\lambda_2\lambda_3^2$
$\lambda_1(\lambda_1-1)(\lambda_2+1)$	$\lambda_1(\lambda_1-1)(\lambda_3+1)$	$\lambda_1(\lambda_2-1)(\lambda_2+1)$	$\lambda_1(\lambda_2-1)(\lambda_3+1)$	$\lambda_1(\lambda_3-1)(\lambda_3+1)$	$\lambda_2(\lambda_2-1)(\lambda_3+1)$	$\lambda_2(\lambda_3-1)(\lambda_3+1)$

Key lemma: $\left\|\mathrm{tr}_{n-k}\rho_\lambda - \mathbb{T}(\sigma(r)^{\otimes k})\right\| = \frac{1}{2}\sum_{\mu\in\mathsf{Par}(k,d)} \dim V_\mu \cdot \left|\frac{s^\star_\mu(\lambda)}{(n\downarrow k)} - s_\mu(r)\right|$

One-and-a-half quantum de Finetti theorems

Matthias Christandl,* Robert König,† Graeme Mitchison,‡ and Renato Renner§
Centre for Quantum Computation, DAMTP, University of Cambridge, Cambridge CB3 0WA, UK
(Dated: October 4, 2008)

When $n-k$ systems of an n-partite permutation-invariant state are traced out, the resulting state can be approximated by a convex combination of tensor product states. This is the quantum de Finetti theorem. In this paper, we show that an upper bound on the trace distance of this approximation is given by $2\frac{kd^2}{n}$, where d is the dimension of the individual system, thereby improving previously known bounds. Our result follows from a more general approximation theorem for representations of the unitary group. Consider a pure state that lies in the irreducible representation $U_{\mu+\nu} \subset U_\mu \otimes U_\nu$ of the unitary group $\mathsf{U}(d)$, for highest weights μ, ν and $\mu+\nu$. Let ξ_μ be the state obtained by tracing out U_ν. Then ξ_μ is close to a convex combination of the coherent states $U_\mu(g)|v_\mu\rangle$, where $g \in \mathsf{U}(d)$ and $|v_\mu\rangle$ is the highest weight vector in U_μ.

For the class of symmetric Werner states, which are invariant under both the permutation and unitary groups, we give a second de Finetti-style theorem (our "half" theorem). It arises from a combinatorial formula for the distance of certain special symmetric Werner states to states of fixed spectrum, making a connection to the recently defined shifted Schur functions [1]. This formula also provides us with useful examples that allow us to conclude that finite quantum de Finetti theorems (unlike their classical counterparts) must depend on the dimension d. The last part of this paper analyses the structure of the set of symmetric Werner states and shows that the product states in this set do not form a polytope in general.

A most compendious and facile quantum de Finetti theorem

Robert König* and Graeme Mitchison†
Centre for Quantum Computation, DAMTP, University of Cambridge, Cambridge CB3 0WA, UK

In its most basic form, the finite quantum de Finetti theorem states that the reduced k-partite density operator of an n-partite symmetric state can be approximated by a convex combination of k-fold product states. Variations of this result include Renner's "exponential" approximation by "almost-product" states, a theorem which deals with certain triples of representations of the unitary group, and D'Cruz et al.'s result for infinite-dimensional systems. We show how these theorems follow from a single, general de Finetti theorem for representations of symmetry groups, each instance corresponding to a particular choice of symmetry group and representation of that group. This gives some insight into the nature of the set of approximating states, and leads to some new results, including an exponential theorem for infinite-dimensional systems.

IV. *A moſt compendious and facile Method for Conſtructing the Logarithms, exemplified and demonſtrated from the Nature of Numbers, without any regard to the Hyperbola, with a ſpeedy Method for finding the Number from the Logarithm given.* *By* E. Halley.

THE Invention of the Logarithms is juſtly eſteemed one of the moſt Uſeful Diſcoveries in the Art of Numbers, and accordingly has had an Univerſal Reception and Applauſe; and the great Geometricians of this Age have not been wanting to cultivate this Subject with all the Accuracy and Subtilty a matter of that conſequence doth require; and

Philosophical Transaction Series I,
vol. 19, p. 58–67, **January 1695**

A most compendious and facile quantum de Finetti theorem

Robert König* and Graeme Mitchison†
Centre for Quantum Computation, DAMTP, University of Cambridge, Cambridge CB3 0WA, UK

In its most basic form, the finite quantum de Finetti theorem states that the reduced k-partite density operator of an n-partite symmetric state can be approximated by a convex combination of k-fold product states. Variations of this result include Renner's "exponential" approximation by "almost-product" states, a theorem which deals with certain triples of representations of the unitary group, and D'Cruz et al.'s result for infinite-dimensional systems. We show how these theorems follow from a single, general de Finetti theorem for representations of symmetry groups, each instance corresponding to a particular choice of symmetry group and representation of that group. This gives some insight into the nature of the set of approximating states, and leads to some new results, including an exponential theorem for infinite-dimensional systems.