Distinguishing Bell states via Bell measurements

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Suppose you are given a <u>single</u> copy of a state $|\psi\rangle$ and told that it is one of the 4 Bell states.

(Q) How does a Bell measurement help you determine which state it is for sure? Doesn't the very first measurement that you do on $|\psi\rangle$ cause a collapse?

(A) No it doesn't cause a collapse. This is the reason why:

Suppose the state $|\psi\rangle$ is the Bell state $|\Psi^+_{AB}\rangle$ &

Say you first ask the question: <u>is the state $|\Phi^+_{AB}\rangle$?</u>

To find the answer, you do a measurement corresponding to the projector $P_{00} = |\Phi^+\rangle\langle\Phi^+|$

Let 'a' denote the outcome of the measurement.

$\text{Prob}(a=00) = \langle\psi|P_{00}|\psi\rangle = \langle\Psi^+|P_{00}|\Psi^+\rangle = 0$

This means you do <u>not</u> get the outcome 00.

$\Rightarrow \text{Prob}(a \neq 00) = \text{Prob}(a \in \{01, 10, 11\}) = \langle\Psi^+|(I-P_{00})|\Psi^+\rangle = 1$

($\because (I-P_{00})$ projects onto the subspace corresponding to the outcomes $a \in \{01, 10, 11\}$)

$\therefore$ Post-measurement state $|\Psi^+\rangle \mapsto \dfrac{(I-P_{00})|\Psi^+\rangle}{\sqrt{1}} = |\Psi^+\rangle$

So the state remains unchanged! There is no collapse.

- Next ask: <u>is the state $|\Phi^-\rangle$?</u> Corresponding projector is $P_{01} = |\Phi^-\rangle\langle\Phi^-|$: $\text{Prob}(a=01) = \langle\Psi^+|P_{01}|\Psi^+\rangle = 0$

$\Rightarrow \text{Prob}(a \in \{10, 11\}) = \langle\Psi^+|I-(P_{00}+P_{01})|\Psi^+\rangle = 1$

$\Rightarrow$ Post-meas. state $|\Psi^+\rangle \longmapsto (I-(P_{00}+P_{01}))|\Psi^+\rangle = |\Psi^+\rangle$

- Next ask: <u>is the state $|\Psi^+\rangle$?</u>; Projector $P_{10} = |\Psi^+\rangle\langle\Psi^+|$

$\text{Prob}(a=10) = \langle\Psi^+|P_{10}|\Psi^+\rangle = 1$ $\boxed{\therefore \text{Inference } |\psi\rangle = |\Psi^+\rangle \text{ WITH CERTAINTY}}$