Heisenberg's Uncertainty Principle

We define the *uncertainty* $\Delta_\psi A$ in a measurement of A on the state ψ by

$$
\begin{aligned}
(\Delta_\psi A)^2 &= \langle (A - \langle A \rangle_\psi)^2 \rangle_\psi \\
&= \langle A^2 \rangle_\psi - (\langle A \rangle_\psi)^2 \,. \qquad (6.21)
\end{aligned}
$$

Note that Theorem 1 implies that the expectation value and the uncertainty are always real, as we would expect if they are physically meaningful.

We can easily verify that $(\Delta_\psi A)^2$ is the statistical variance of the probability distribution for the possible outcomes of the measurement of A on ψ, and $\Delta_\psi A$ is the distribution's standard deviation.

I will put this slide up (please verify.)

We can verify this directly for operators with discrete eigenvalues, and also for position.
We take it as a definition (which can be justified with more work) for general operators.